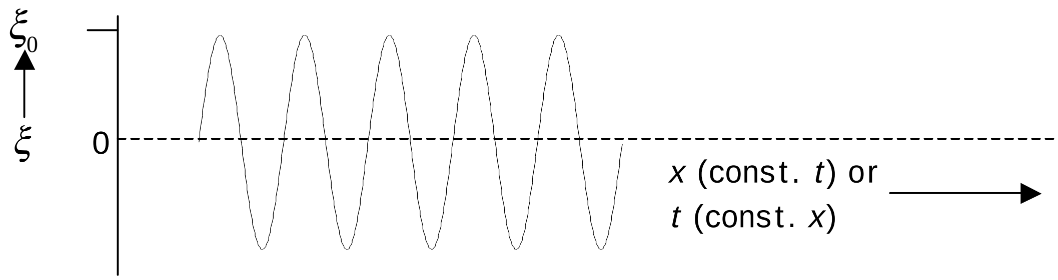
WAVE-PARTICLE DUALITY of LIGHT and MATTER

(A) Light (electromagnetic radiation)

Light as a wave

For now we neglect the polarization vector orientation
Propagating in x-direction:

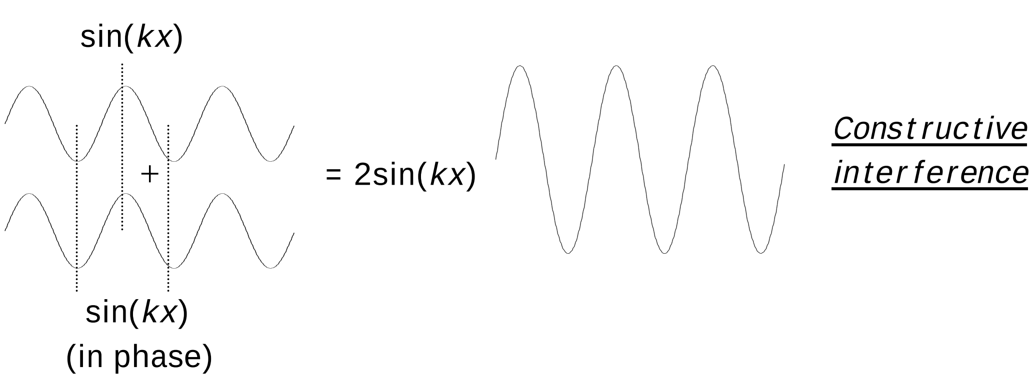


$$\xi(x,t) = \xi_0 \sin\left[\frac{2\pi}{\lambda}(x - ct)\right]$$

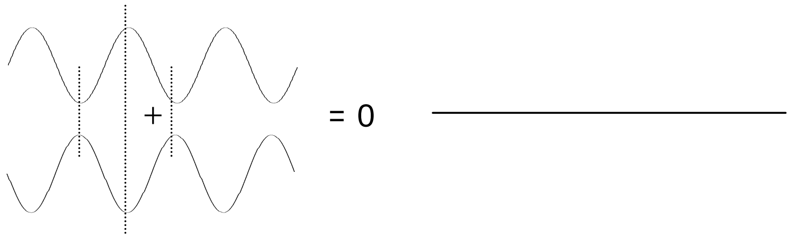
Define $k = \dfrac{2\pi}{\lambda}$ ("wavevector" magnitude)

At some fixed time, say $t = 0$, can write simply $\xi(x, t = 0) = \xi_0 \sin(kx)$

Superposition principle



sin(kx)

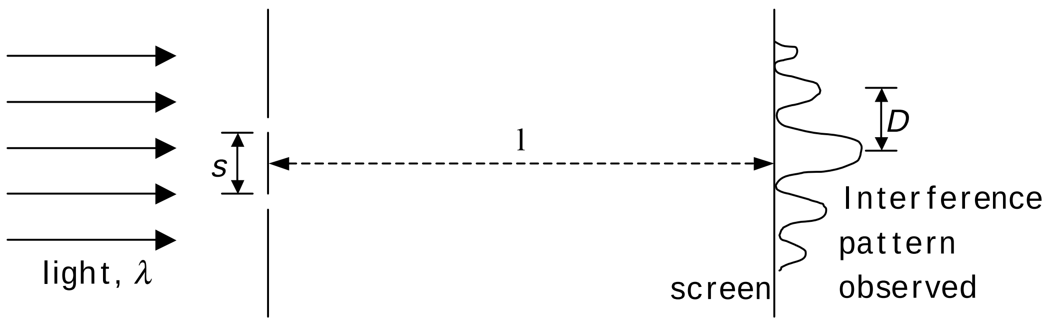


sin(kx + π)
(out of phase by λ/2)

<u>*Destructive interference*</u>

<u>This leads to many interference phenomena</u>

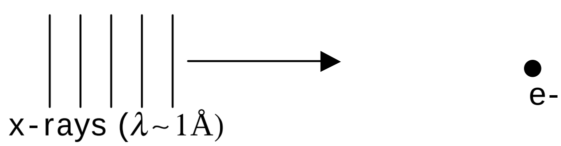
Young's 2-slit experiment



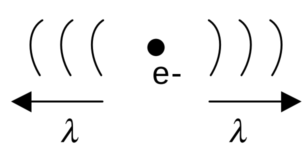
$$\frac{D}{l} = \frac{\lambda}{s} \quad \Rightarrow \quad \boxed{D = \frac{\lambda l}{s}}$$

<u>Light as a particle</u>

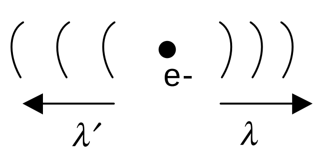
Compton exp't



If just a wave, expect light to scatter off electron



Experimentally:



The backscattered wave is red-shifted ($\lambda' > \lambda$), i.e. less energy/photon.

$$E' = \frac{hc}{\lambda'} < \frac{hc}{\lambda} = E$$

Energy (and momentum) transferred to the electron.
Need relativistic mechanics to solve

$$p = \frac{h\nu}{c} \left(= \frac{h}{\lambda} \right) \quad \text{for the light}$$

Light is a particle with energy

momentum $p = \dfrac{h\nu}{c}$

> Light can behave both as a wave and as a particle!!
> *Which aspect is observed depends on what is measured.*

(B) Matter

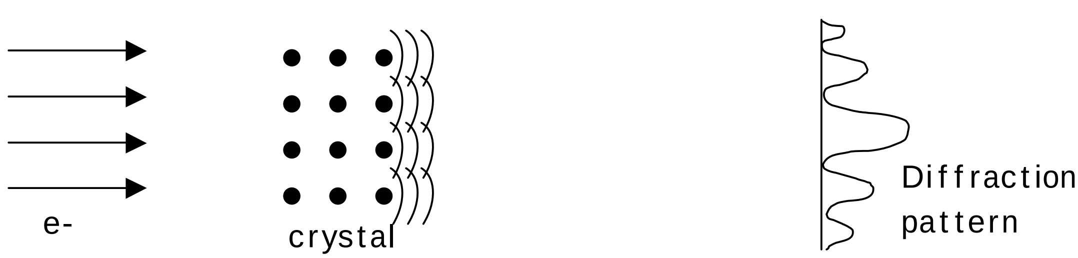
Matter as particles $\Rightarrow$ obvious from everyday experience

Matter as waves (deBroglie, 1929, Nobel Prize for his Ph.D. thesis!)

Same relationship between momentum and wavelength *for light and for matter*

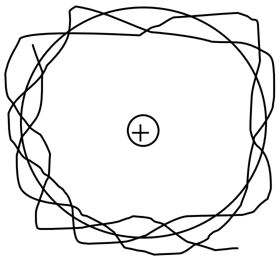
$$p = \frac{h}{\lambda} \quad \Rightarrow \quad \boxed{\lambda = \frac{h}{p}} \quad \equiv \textit{de Broglie wavelength}$$

Amazing notion! But wavelength only observable for microscopic momentum

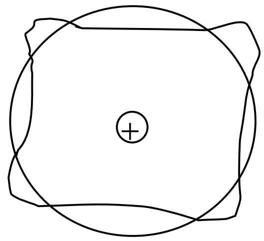


Consequences (I)

(1) on Bohr atom



If e- wave does not close on itself, eventually destructive interference will kill it!



If e- wave does close on itself, then constructive interference preserves it.

Criterion for stability:

$$2\pi r = n\lambda = \frac{nh}{p} = \frac{nh}{mv}$$

$$\text{or} \quad mvr = \frac{nh}{2\pi} = n\hbar$$

$$\Rightarrow \quad \boxed{\ell = n\hbar}$$

As Bohr had assumed angular momentum is quantized!