6.581/20.482

LECTURE 16: STEADY-STATE PROBLEMS

TUESDAY 11 APRIL 2006

SIMPLIFIED SYSTEM

$$\frac{d}{dt}\vec{x} = A^{(1)}\vec{x} + A^{(2)}(\vec{x}\otimes\vec{x}) + B\vec{u}$$

STEADY STATE

Solve $\underbrace{A^{(1)}\vec{x} + A^{(2)}(\vec{x}\otimes\vec{x}) + B\vec{u} = 0}_{F(\vec{x})=0}$

NEWTON'S METHOD

Guess at $\vec{x}^0$

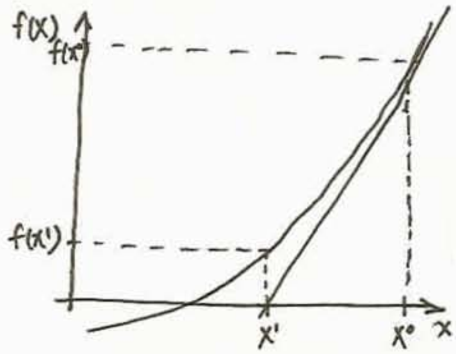
Evaluate $F(\vec{x}^0) \neq 0$

$\simeq F(\vec{x}^*) + J_F\underbrace{(\vec{x}^0 - \vec{x}^*)}_{\Delta x}$

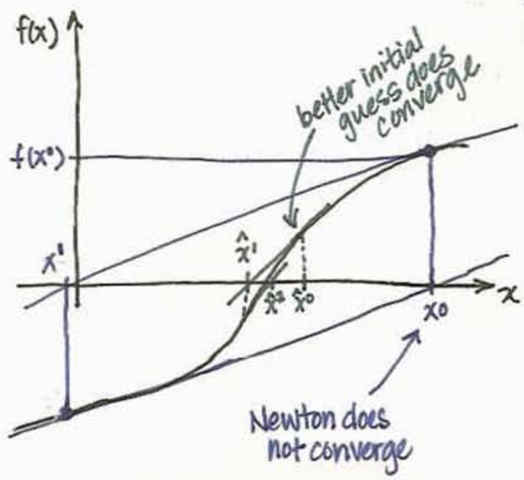
Use Approximation

$J_F(\vec{x}^0)\Delta x = -F(\vec{x})$

update $\vec{x}^1 = \vec{x}^0 + \Delta x$



$$J_F(x) = A^{(1)} + A^{(2)}(\vec{x}\otimes I + I\otimes\vec{x})$$



NEWTON PROPERTIES:

1. Converges if you are close enough
2. Converges very fast

} $J_F(\vec{x})$ is nonsingular

Original F:

$F(\vec{x}) = A^{(1)}\vec{x} + A^{(2)}\vec{x}\otimes\vec{x} + B\vec{u}$

Simpler F: $\vec{u}=0$

$F(\vec{x},0) = A^{(1)}\vec{x} + A^{(2)}(\vec{x}\otimes\vec{x})$

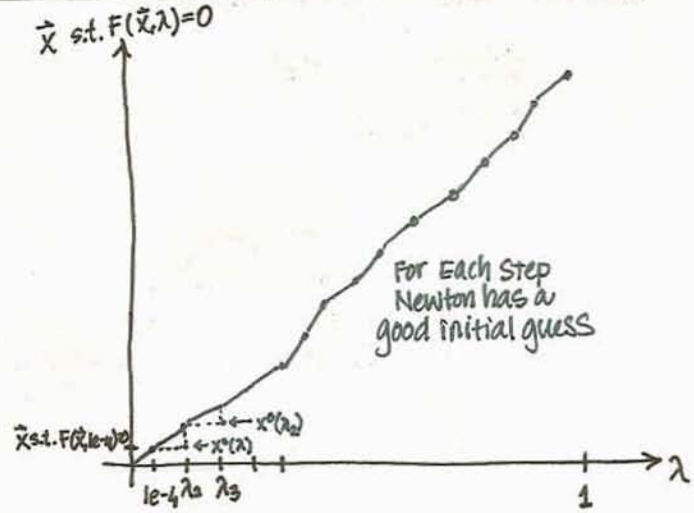
$\vec{x}$ s.t. $F(\vec{x},0)=0$ $\quad \vec{x}=0$

Continuation scheme:

$F(\vec{x},\lambda) = A^{(1)}\vec{x} + A^{(2)}\vec{x}\otimes\vec{x} + \lambda B\vec{u}$

$F(\vec{x},0) = A^{(1)}\vec{x} + A^{(2)}\vec{x}\otimes\vec{x} \Rightarrow \vec{x}=0 \;\&\; \lambda=0$

(original problem: $F(\vec{x},1)=0$)



WANTED INFORMATION:

- How many stable steady-states
- What are the stable steady-states

NEWTON GIVES:

- Maybe one steady-state

Example 1.

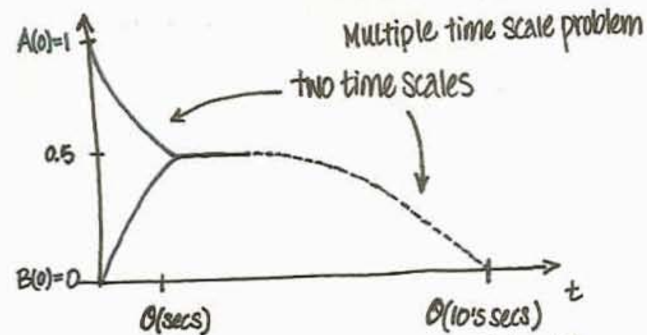
$$A \underset{1}{\overset{1}{\rightleftharpoons}} B$$

A degrades 0.1, B degrades 0.1

$$\frac{d[A]}{dt} = -1.1[A] + 1[B]$$

(degradation)

$$\frac{d[B]}{dt} = 1[A] - 1.1[B]$$



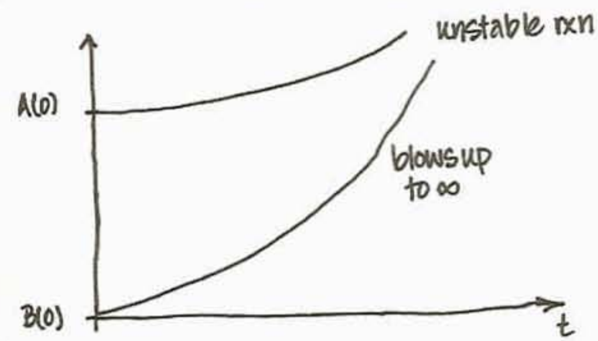
Example 2.

$$\overset{3}{\circlearrowleft} A \underset{1}{\overset{4}{\rightleftharpoons}} B$$

$\frac{d}{dt}[A] = (-4+3)[A] + 1[B]$ (Not tracking every species)

$= -1[A] + 1[B]$

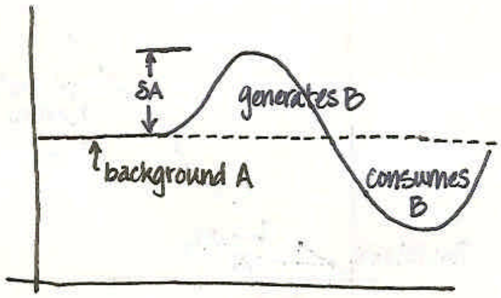
$\frac{d}{dt}[B] = 4[A] - 1[B]$

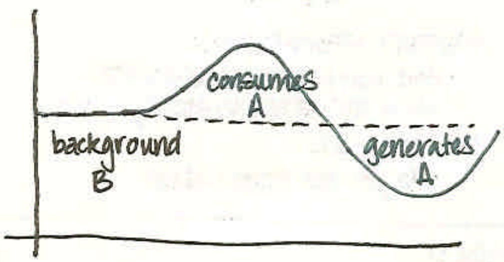


Example 3. Oscillatory System (nonphysical)

$$\frac{d}{dt}[A] = \quad -1[B]$$

$$\frac{d}{dt}[B] = +1[A]$$





$$\frac{d}{dt}\vec{X} = \begin{bmatrix} -1.01 & 1 \\ 1 & -1.01 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$A \underset{1}{\overset{1}{\rightleftharpoons}} B$, $A \xrightarrow{0.01}$, $B \xrightarrow{0.01}$

$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_i = -2.01, -0.01$

Fast Mode, Slow mode

$$\frac{d}{dt}\vec{X} = \begin{bmatrix} -1 & 1 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$A \underset{1}{\overset{4}{\rightleftharpoons}} B$, $A \xrightarrow{3} A$

$\lambda_1 = -3$ $\underbrace{\lambda_2 = 1}_{\text{unstable}}$

$$\frac{d}{dt}\vec{X} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$\delta A \; ? \; \delta B$

$\lambda_1 = j$ $\lambda_2 = -j$

oscillatory soln w/o decay ⇒ purely imaginary eigenvalues

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