

Q4. How has research into shape grammars evolved since the 1970s?

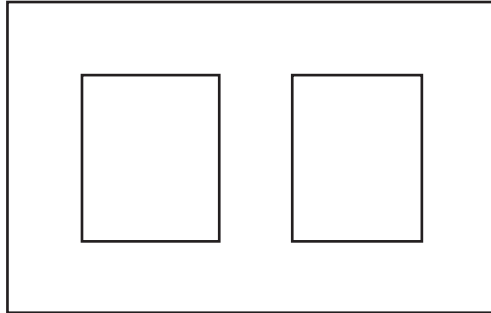
A4. Shape grammar research has taken off in many directions. There's a lot of work – some of my own but mostly of others – that uses shape grammars to investigate style and typology in art and architecture – what's a Palladian villa, what's a courthouse, etc.? – and for forgery. Shape grammars have also been used creatively both in architecture and in product design – their promise in practice is immense, waiting for designers to risk it and try them out. There's a striking body of work, too, that shows how visual calculating in shape grammars can be used to focus art and design education. Some of this depends on recent theoretical results that define “schemas” as heuristics for rules of varied kinds and establish their relationships. The rudiments are too tempting to leave out. I promise – the details are painless in a short diversion that's scenic and worth the time and a few words. In fact, one of the examples I like to use for schemas prefigures what I want to say about how insight and imagination work, and how they're included in shape grammars. A schema

$$x \rightarrow y$$

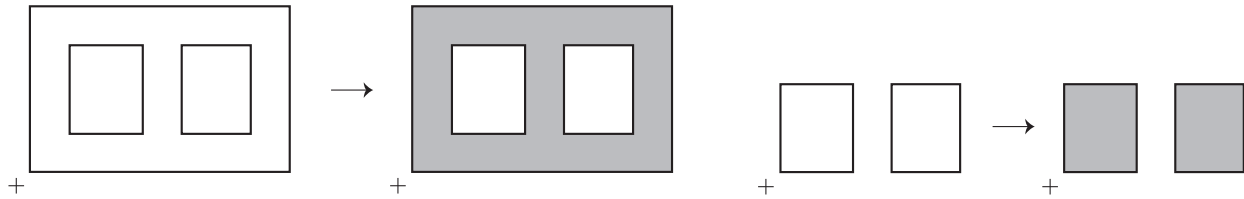
has variables, here x and y , that are assigned shapes as values to define a set of rules. Primary schemas for parts, transformations, and boundaries imply others, with subsets, copies, inverses, adding, composition, and Boolean expressions. For example, adding the schema $x \rightarrow x$ for identities that's a subset of the schema $x \rightarrow \text{prt}(x)$ for parts, and the inverse of the schema $x \rightarrow \text{b}(x)$ for boundaries explains the artful workings of the coloring book schema

$$x \rightarrow x + \text{b}^{-1}(x)$$

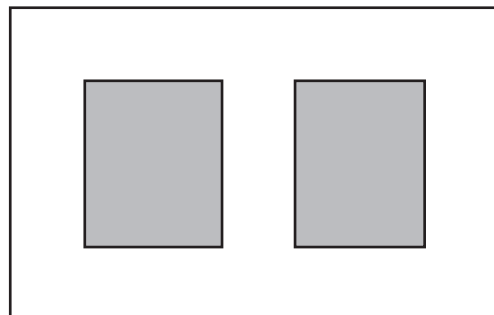
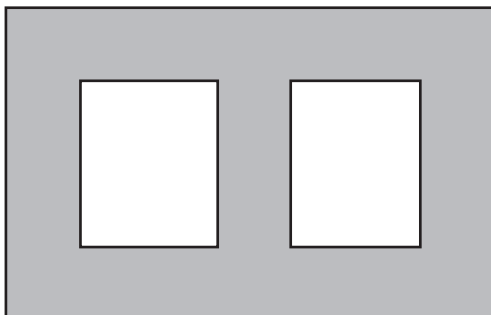
that fills in areas and saves their outlines – for alternate values of the variable x in a building plan, there are walls or poché, or rooms and spaces. For this shape



the two rules in $x \rightarrow x + b^{-1}(x)$ are these

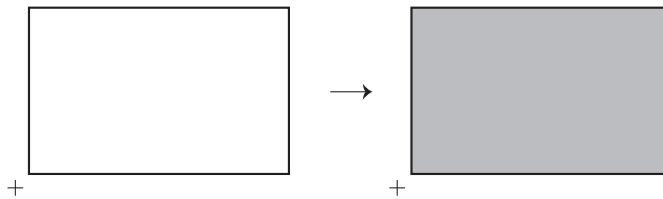


to fill in areas with grey. This gives



(In oddly artsy/geeky moments – sometimes, I can't help myself – I put the composition $b^{-1}(\text{prt}(x))$ in the schema $x \rightarrow x + b^{-1}(\text{prt}(x))$, because there's a single value for x , namely, the shape itself. The schema switches walls and rooms for two distinct parts, and there are five other parts for which the inverse boundary schema is defined. The coloring book schema is a subset of $x \rightarrow x + b^{-1}(\text{prt}(x))$; both go nicely with weights, and are similar versions of the summation schema in the next parentheses.) I can use weights to vary shades of grey, so that adding the grey in a rule to any grey in a shape makes the grey in the shape lighter.

My twin rules and this one



also in the coloring book schema, define indefinitely many shapes, including these two



(There are lots of nifty possibilities – tellingly, r is part of n – not as a letter or symbol but as a shape – so the rule $n \rightarrow r$ is in $x \rightarrow \text{prt}(x)$, ready to try in A3 for new meaning. A number of years ago, I used the rule $h \rightarrow r$ to show that cheating includes creating, as changed includes charged. This changed-charged cheating to empower artist and novice alike with a creative method that actually works, and that's easy to use. Everyone does it, even if few will ever say so – it's just not encouraged. Cheating and plagiarism are an

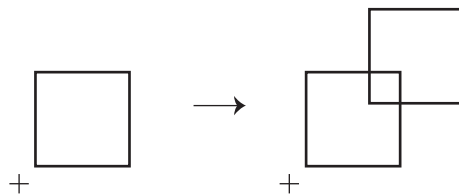
unfailing source of delightful things. Copying is key in both education and practice – inventing is seeing. L. B. Alberti fixes the original locus of seeing in *De statua*, when artists, “diligently observing and studying,” were led to find in nature, “in a tree-trunk or clod of earth,” outlines to trace and complete in drawings, pictures, sculpture, etc. – by adding to, taking away, or otherwise supplying what seemed lacking. What would Simon say, and Sutherland? This kind of creative copying – but all copying is creative – extends neatly throughout art and design from tree-trunks and the like to pictures whether finished or not, mine or others, to whatever I choose to see in a beautiful form or a Rorschach test; it’s what happens in visual calculating in shape grammars, without any fuss. In schemas, rules can be described in sums of changes to shapes and their parts – to copy in this way and that for something fresh and new. Try the schema

$$x \rightarrow \sum F(\text{prt}(x))$$

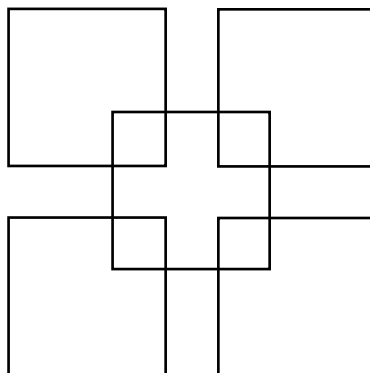
on a given shape C , where the variable x is C or anything in it, and F tells what rules apply to parts of x , say, rules in the schema $x \rightarrow t(x)$ for transformations or in the coloring book schema. All of x is taken away in this process, as its parts – any of them, even multiple times – are replaced with new ones. Every rule $A \rightarrow B$ is in the schema – for shapes A and B , $\text{prt}(x) = x = A$, and $F(A) = B$. This empty inclusiveness aside, the schema suggests rules and helps to classify them. Rules are global and local at the same time, and go for large features and small details in open-ended improvisation. The schema is good for rules that parse or divide shapes in graphs, trees, topologies, etc. It shows how rules conflate standard types of symbolic calculating – context free vs context sensitive, sequential vs parallel – when shapes fuse to blur distinctions for units/symbols. More generally, the schema accents the inconstant relationship between shapes and symbols – separate processes are distinguished in the individual terms $F(\text{prt}(x))$ that aren’t fixed in the rules the schema defines, or in the shapes these rules produce. The schema describes rules to make shapes that needn’t be described in the same way – I can always see something new. Just as it is for units/symbols, distinctions in the schema are lost when shapes fuse; they aren’t kept to block my way. They may be memorable and true, but so what – I’m free to change my mind at will. That’s why embedding is key and why shapes fuse, and why rules are the way they are. Let’s try the schema to see exactly how it goes – to see it is to understand it. If x is the square



and $\text{prt}(x)$ is x four times, then for F given by copies of the addition rule

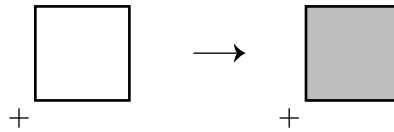


in the schema $x \rightarrow x + t(x)$ that combines an identity and a transformation – here, a translation on the diagonal of a square – I get the shape



How many squares are there now? What's next, doing this again for the four small squares that aren't described in the addition schema I used to define F ? How about for the extra big squares, or for different combinations of big and small squares? There are many ways to see this, with squares and without. And

what happens if I decide to include one more rule



in the coloring book schema, and grey gets lighter as it's used? This is just warming up. Still, enough is enough – technical agility isn't the goal, rather something palpable and intuitive.) Whenever they're used, shape grammars incorporate insight and imagination in visual calculating. Sometimes, it's helpful to look at insight in terms of gestalt effects, for example, figure and ground reversal in building plans, when walls and rooms switch back and forth, in ice-rays and other Chinese lattice designs, and in painting and sculpture – for example, in Ellsworth Kelly and David Smith. (Marjorie Garber does the same for poems and language broadly in her stern review of George Lakoff's and Mark Turner's use of "mappings" for metaphor. Mappings confuse meaning and structure, to "ground" metaphor in "conceptual domains" that are coded in "schemas." Garber rejects mappings and schemas because they limit how metaphor works and narrow its expressive sweep. To keep metaphor open-ended, she turns to Gestalt psychology and figure-ground reversal, in the way von Neuman puts the Rorschach test beyond calculating with visual analogies – in fact, spatial relations for shapes show what schemas do and why mappings must invariably fail. However, the psychology of figure-ground reversal or the Rorschach test isn't required to explain metaphor. Cognitive science is mostly irrelevant – shapes are ambiguous no matter the range of physical parameters. This doesn't say much about what I see. Wilde sticks to music, and pictures and poems; he asks simply for a beautiful form. "Beauty is the symbol of symbols" –

Beauty has as many meanings as a man has moods. Beauty is the symbol of symbols.

Beauty reveals everything, because it expresses nothing. When it shows itself, it shows us the whole fiery-coloured world. (*Intentions*)

For Garber, the Rubin vase is the “figure for *figure*,” that is to say, for metaphor in poems, etc. Every metaphor is a beautiful form or a Rorschach test. Reading and rereading rely on insight – “All language is figure, and figuration: it is the idea that we can see through language [in rich and changeable ways] to encounter the real.” And what’s real in figure and metaphor hinges on the fickle results of perception and personal method, on ambiguity and the arc of negative capability. This isn’t for mappings and schemas, where the real is grounded in a conceptual domain with constant structure – meaning for pictures and poems isn’t expected to change, but then what? Schemas in shape grammars are otherwise; shapes have no structure in themselves – they alter with personality and mood.) I bank on embedding for this, and for Alberti’s locus of seeing “in a tree-trunk or clod of earth” (Rorschach test) and what it implies – as Shakespeare’s Theseus fears, “a bush supposed a bear;” to Alice’s surprise, a Cheshire cat that appears and disappears at will (whose?), the single all and the many not all in the tangled branches of a tree and their interstices; and of course, in Barfield’s chaotic and inane world, a dog with spare limbs at play in a vegetable marrow (coadunate flora and fauna, but how did our ancestors cope with so many shifting shapes in dense forests and wild woods, and how do we manage with the relentless ambiguity in vast and sprawling urban spaces?). Shakespeare’s “strong imagination” conjures “such tricks,” in pictures and poems – and throughout art and design. Questions of plagiarism and fussy exegesis aside, this strikes me as part and parcel of Coleridge’s extraordinary “esemplastic power” – “the *mode* of its operation” is to “shape into one” for new perception in the embed-fuse cycle in shape grammars that I mentioned earlier in A3. Esemplastic power is the source of imagination and yields it fully – imagination

dissolves, diffuses, dissipates [boundaries, to fuse memoryless/structureless wholes], in order to re-create ... It is essentially *vital*, even as all objects (as objects) [especially the ones in computers, and in object-oriented programming and BIM] are essentially fixed and dead. (*Biographia Literaria*)

This puts objects (shapes and things) in motion, as it activates seeing. Parts alter freely – they fuse and re-divide in sense experience, in no way ever fixed and dead. There’s the pulse of life. (Whether this ongoing process repeats known divisions or lets in new ones is mostly irrelevant – because parts fuse with no

memory of their boundaries, the same kind of calculating is a must either way. But not everyone is easy with this. It would seem that Barfield reaffirms known divisions in final participation and regrets the extravagance of new divisions when participation is original. Other distinctions are out, as well. In particular, whether to fuse and divide is conscious or unconscious doesn't matter – there's no difference in calculating. In unguarded moments, though, I use the Cartesian product of the pairs known/new and conscious/unconscious as a rough-and-ready way into a taxonomy for Coleridge, Barfield, and no doubt, others. When my guard is down, I'm liable to see just about anything. And first impressions are effective more times than not – it's amazing how strange things can be in art and design. Figuration in varied forms is key – caricature, distortion, exaggeration, metaphor, etc. Maybe the soul is imagination. More practically, I find simplifying to the limits of recognition a rich way to unpack any unfamiliar relationship. For example, neither Eastern thought nor Western sophistication seems particularly well disposed to imagination – it's mostly a tempting nuisance. Everything in the East is one, with no compensating emphasis on embedding or individuation; and in the West, atoms merely combine and never fuse. That's how it seems to me. But East and West merge and dissolve, coadunate in shape grammars – in every pulse of the embed-fuse cycle. The trick is nearly always the same, to keep this series in constant motion, so that there's never a pause.) Without embedding and the insight it brings, it's hard to go on to anything new; without imagination, “that synthetic and magical power,” the critical spirit soon withers and dies. There's more of this elsewhere – in literary criticism. I. A. Richards traces Coleridge's imagination in much the way I do – “earlier acts of perception [that] ... have come into being, been formed, by earlier acts of Imagination [are] ... reformed ... integrated, co-adunated into new perception.” In shape grammars, constituent parts (units) needn't limit what I see – I can embed two squares in four triangles when they fuse. And in exactly the same way for poems, “*Words [triangles] are not necessarily the units of meaning [squares].*” For all of us, this is a real possibility – pictures are whatever we see (embed) in them now and change whenever we look again; poems mean whatever we read and talk about now and change as we go on reading. (Some today prefer misreading, although this is likely a mistake. Does anyone ever say mis-seeing? Suppose we did – what difference would it make? Isn't mis-seeing, seeing and so, believing?) New perception never ends; what we find in pictures and poems isn't final. Richards is especially keen on this, as it influences our established beliefs and values –

[N]either Coleridge's grounds for [imagination] nor his applications of it have as yet entered our general intellectual tradition. When they do, the order of our universes will have been changed. (*Coleridge on Imagination*)

In Coleridge's esemplastic power, there's a kind of active "projection" – we make new parts to embed in things that fuse. (The alternative is to talk about parts that emerge. Many see this when a pair of discrete squares on a common diagonal intersect to produce smaller squares in their shifting interstices – this is McCarthy's lines and tellingly, embedding, too.) It's an inspiring process, but for "convenience," Richards "dispenses with [it]." Not "denying the validity of [Coleridge's] projective account," he turns instead to practical criticism and to that with which the critic is most concerned – a structural analysis of "differing types of interaction between parts of a total meaning." In the terminology of math and computers, he settles for a graph, very likely directed with strongly connected components. Of course, graphs alone are never the source of imagination in pictures and poems or in any results in art and design; they're merely an effective way to describe and measure given aspects and properties that are determined independent of graphs. However, the method is seductive – few resist it – even if it requires a steady hand and often leads to ruin as an easy substitute for imagination. But this isn't a problem for Richards – he always shoots for what's important and isn't known to miss. He values imagination highly – "Doubtless the ideal case of Imagination is rare." I wonder why. Is there something about imagination in itself that makes this so, or is it simply what graphs and the lure of convenience imply? It's pretty hard to tell outside of shape grammars – they assimilate Coleridge indifferently without any fuss, as they fill in the details of Richards's total meaning in a reciprocal process with description rules to put in parts and to fix their interactions, in retrospective graphs and topologies. In shape grammars, insight and imagination are unavoidable; they come in automatically as rules are tried. Imagination works recursively, rule after rule for all to see. It isn't so much that imagination is rare, but that we tend to suppress it, preferring the truth (security) of memory (nostalgia) to the risk of new perception. Imagination unfolds neatly in the embed-fuse cycle. I can show exactly where it works and how, and why anything less muffs its tricks, graphs and topologies included – although many distrust my endless run of examples that must be drawn and seen.² Nearly everyone I know, computer scientist and not, believes that symbolic calculating – combining given units (atoms, bits, building blocks, objects, primitives, simples, etc.) – covers all the bases. My neighbors in Brookline tell me the same thing – the famous biologist a few houses down on the right embraces "BioBricks" for Lego-like design in synthetic

biology, while the rigorous computer scientist on the next street to the left lectures me zealously about the atoms of computation and their importance for recursion. This seems perfectly correct, especially when you're outflanked on two sides. Symbolic calculating is the final standard, steeped in structure and averse to any ambiguity; there's no room for question or doubt. Its paired loci span everything with complete certainty – in axiomatic logic, and in Bayesian statistics and deep learning. In a funny way, both trace the identical path. And both fall short of insight and imagination that are crucial to keep art and design creative, and really, insight and imagination are necessary for science and engineering, too. The Gestalt psychologist Wolfgang Köhler and the cyberneticist Warren McCulloch were surely the first to take this seriously around 1950, for Norbert Wiener's statistical version of cybernetics in which feedback is key – and it goes equally for McCulloch's groundbreaking calculus of nervous activity (neural nets) in which you can calculate anything you can describe in words, or likewise, with numbers and data. No surprise, McCulloch touts every bit of cybernetics and calculating (computers and AI) with unreserved enthusiasm and unflagging confidence –

But the problem of insight, or intuition, or invention – call it what you will – we do not understand, although ... that is the problem I should tackle. ("What Is a Number, that a Man May Know It, and a Man, that He May Know a Number?")

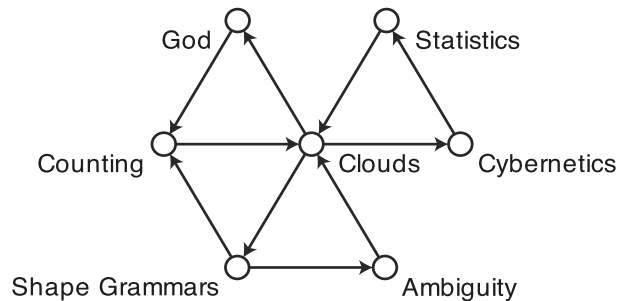
(In *Cybernetics*, Wiener recites a little, children's song about how God with matchless power can count the stars in heaven and the clouds in the sky, and knows when all of them are there. The translation of the original German runs so –

Knowest thou how many stars stand in the blue tent of heaven? Knowest thou how many clouds pass over the whole world? The Lord God hath counted them, that not one of the whole great number be lacking. (*Cybernetics*)

Wiener uses the song to split science into expert disciplines, contrasting astronomy and meteorology – definite things like stars and their constellations are ready to count one by one and vary with unerring regularity, while indefinite things like clouds with fuzzy boundaries and random variations are secure in data

and statistics. The song also defines a distinct choice in shape grammars. First there's identity – calculating with visual analogies or spatial relations, where counting is one by one with the members of a set, or points that add up like stars, and shapes are sets/constellations. And then there's embedding and things fuse – calculating with shapes that are like clouds to see/find faces, animals, and sundry images and figures à la Alberti and the Rorschach test, where ambiguity reigns, and counting and statistics are unstable and break down as boundaries dissolve and re-form, free and unrestrained. Incredibly, Wiener's song puts God, cybernetics, and shape grammars together; the three merge as one in a comprehensive verse – the song is a beautiful form. Its lines trace counting, statistics, and ambiguity. To God, counting is all it takes to survey all there is. Clouds are as definite as stars – everything in heaven and on earth can be tallied in a perfect census. Lacking God's power to count fuzzy things, cybernetics turns to data and statistics to fix the locus of “*control and communication in the animal and the machine.*” And shape grammars dissolve and diffuse divisions to fuse known parts into one – ambiguity/embedding supersedes counting to include art and design, and the critical spirit in new perception. Of course, taxonomies aren't always entirely straightforward. Sometimes, three make an asymmetrical two – counting and statistics go for certainty, while ambiguity guarantees the exact opposite. Is there a convenient way to track all of the meaningful paths in this mass of interactions? Maybe it's time for a Richards-like graph, if only to show how worthwhile this is, even for a little song. My graph has 26 vertices and too many edges to ever count, although once shape grammars are added in, it's clear that calculating connects to more than God or science – I guess this spells the end of physics and a theory of everything. Cool, except the complete graph won't do for eye and hand. With its scattered vertices and crisscrossing edges, paths are hard to untangle and trace. It takes a computer to map them out and to tidy up. Maybe a simple subgraph merits a serious try – at the heart of my graph, there's a subgraph in which clouds connect God and counting, cybernetics and statistics, and shape grammars and ambiguity in three triangular cycles. These fit in a regular heptagon, with clouds at their common vertex. But this isn't the end of it – for shape grammars, there's embedding and ambiguity plus identity and counting, and so an extra edge and another triangle. Putting the new edge in place between shape grammars and counting, makes it tempting to play God with visual analogies, and computers and AI – invariably, this spells disaster. It makes better sense to enhance the symmetry of my heptagon in a triaxial pinwheel, with clouds at the center. Now I can see clearly how Wiener's song works – without overwhelming detail, in the *CliffsNotes* version that's from my original graph. There are meaningful

paths to trace, in four equilateral cycles, the three in the pinwheel going counterclockwise, and another one going the other way around. Maybe there's really something to this. It feels objective; finally, it's getting it right. This is one for the ages –



Graphs seem to work – however, the method from song to graph to subgraph is surely incomplete. What good are graphs or mappings or whatever structures I use, if I can change my mind about what I see at any time, and want to factor this in? But I can do this in shape grammars in the embed-fuse cycle – in fact, as a magical extension of feedback in cybernetics, so that sense experience is unrestricted with nothing set in advance. Who would have ever thought that clouds held an alternative to God and Wiener's cybernetics in shape grammars, when there's embedding and shapes fuse? Isn't all of this proof positive for visual calculating, that there's something more besides counting and statistics in open-ended ambiguity? In shape grammars, ambiguity is the shifting frontier of extravagant delight – the Vitruvian aesthetic third, above and beyond all use. Calculating is for science, and for art and design, too.) Solving McCulloch's problem is a lot harder than it looks at first blush. Progress is elusive – maybe a shape isn't a number or data, or maybe insight isn't to be found anywhere in descriptions (visual analogies, graphs, and mappings). Seventy years later and still counting, McCulloch's problem has become a lasting constant in computer research and AI, older than Moore's law and well outside its scope. No doubt, there's always the chance of a big breakthrough. IBM's Watson is hyped in this way. Nonetheless, not everyone is impressed with its skill, even with its crushing win at Jeopardy. David Gondek, a leader on the Watson research team, is quick to note that scant of Watson's mega power for calculating/thinking matters – "Insight remains handmade." Moreover, it's exactly the same for Deep Blue, AlphaGo, and deep learning programs of every kind. Insight is ours to have, and to impart in retrospect – that vital switch in perception that opens up new ways to go

on, independent of symbolic calculating and the vast resources of computers. (Aren't the rules in shape grammars for insight, and how to go on?) This is good news to many – that computers aren't creative and aren't likely to be anytime soon, maybe not at all. Creativity is forever out of reach, as long as everything is kept only to a number or data. The food columnist and blogger David Sax agrees; he follows his own formula/recipe, blending in imagination and extra seasoning, as he stirs sensibly with eye and hand, unencumbered by mind in computer technology –

Creativity and innovation are driven by imagination, and imagination withers when it is standardized, which is exactly what digital technology requires – codifying everything into 1s and 0s, within the accepted limits of software. (*The Revenge of Analog: Real Things and Why They Matter*)

But try to say more without shape grammars and Coleridge. (In the past, I've fooled with C. S. Peirce's "abduction" as the instinct to imagine correct theories and hypotheses from scattered data, and a nice way to be original. It strikes me now that abduction aims for noticeably less than insight and imagination in shape grammars. With its focus on verified facts and units, abduction misses what's quick to see in art and design.) STEM subjects are a must just about everywhere in education, usually at the expense of the humanities, the arts, physical fitness, and other subjects that don't divide things into units. But without insight and imagination in art and design – without shape grammars – STEM may seem meaningless, mere counting waiting for a reason why. It's textbook, rote, mechanical, and dull, with scant opportunity to see past visual analogies, to be creative, or to find out how. To use Coleridge's famous distinction, it's "fancy" or combining units from memory – "counters," and "fixities and definites" – and testing results in ranges of choice, rather than imagination that's infused with new perception. (Henri Poincaré's tale of mathematical creation is pure fancy. "Atoms" are held in memory or a given cache – "motionless ... hooked to [a] wall," they're taken down and put in "swing" to "flash in every direction," so that some of "their mutual impacts may produce new combinations." The "rules" for choosing atoms and their combinations, however, are "felt rather than formulated" – really, there are no rules just a feeling. Does this kind of intuition traverse fancy, for unknown regions where "to invent, one must think aside," or to put thinking aside, one must see askance? Salvador Dali urges something similar in his "paranoiac-critical method" that risks the unplumbed depths of imagination, seeing in as many outré ways as possible, full of genius and folly alike. Maybe shape grammars

are Dali's dream and the paranoiac's delight. For Paul Valéry, poetic invention is also combination and choice, although unequally – “genius is much less the work of the first ... than ... the second.”) Some argue that fancy and imagination are arbitrary, a distinction without a difference – they vary only in degree, and not in kind. The distinction can be drawn either way in shape grammars, with marvelous indifference. (Likewise, it's easy to separate Poincaré and Valéry from Dali in shape grammars, or to keep a trio that performs fluidly as one.) Many in education, though, settle willingly for visual analogies to make new perception unnecessary. Fancy and imagination are distinct, so that you see exactly what you're taught by rote, and never in your own way. It seems worth it, if it means a good job. The hard questions are simply ignored – what are units for, where do they come from, and how do they change? Visual calculating in shape grammars answers these questions, and moves on. And finally, there's the technical stuff on computer implementations for shape grammars– in the cases where the elements in shapes are given in different analytic expressions, for example, linear or quadratic polynomials. This takes some math, but it's totally fascinating. It's amazing how much needs to go into computers to get them to see in the way that shape grammars do without visual analogies, for schemas and rules to work easily with parts that can change without rhyme or reason, with transformations of varied kinds, and with shifting boundaries. This is indispensable – until I actually try a rule, there's no way of telling what I'll see next. With embedding, I'm free to put whatever I wish in shapes, and there's no trace (memory) of this if I decide to look again. Somehow, I've circled right back to Wilde's beautiful form and to von Neumann's Rorschach test – to insight and imagination, and the embed-fuse cycle that keeps shapes and shape grammars forever vital. And surely, this is the heart of the matter. The conclusion is inevitable, yet to many it may come as a surprise – shape grammars are a more generous standard for calculating than Turing machines and computers. The proof is evident in art and design, and pictures and poems, and for Coleridge farther out than fancy goes in imagination's magical realm of esemplastic power.

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