MIT 16.90 Spring 2014: Problem Set 6

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Due: Monday April 7, in class

Problem 6. 1 *Fourier Analysis of the semi-discrete form*

Consider the following one-dimensional PDE

$$\frac{\partial U}{\partial t} + \frac{\partial U}{\partial x} = \frac{L}{Re}\frac{\partial^2 U}{\partial x^2} \tag{1}$$

where L and Re are positive constants. We will assume periodic boundary conditions and a fixed finite difference grid in the domain of $x \in [-L/2, L/2)$, with a uniform node spacing Δx.

- Is the analytical solution bounded? Why?
- Apply central difference approximation for both the first and second order derivatives. Use these approximations to write Eqn. 1 in semi-discrete form (i.e. discrete in space but continuous in time).
- Writing the solution at grid point j as $U_j(t) = \hat{U}_m(t)e^{i\beta_m j}$ where $\beta_m = k_m \Delta x = \frac{2\pi m}{L}\Delta x$ and m is the mode number, perform a Fourier analysis of the semi-discrete form.
- For a forward Euler time discretization, determine the maximum allowable time step Δt for stability given a fixed Δx, L and Re.

16.90 Computational Methods in Aerospace Engineering

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