16.885J/ESD.35J
Aircraft Systems Engineering

Introduction to Aircraft Performance and Static Stability

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Today's Topics

- Specific fuel consumption and Breguet range equation
- Transonic aerodynamic considerations
- Aircraft Performance
  - Aircraft turning
  - Energy analysis
  - Operating envelope
  - Deep dive of other performance topics for jet transport aircraft in Lectures 6 and 7
- Aircraft longitudinal static stability

Thrust Specific Fuel Consumption (TSFC)

- Definition: $$TSFC \equiv \frac{\text{lb of fuel burned}}{\text{(lb of thrust delivered)(hour)}}$$
- Measure of jet engine effectiveness at converting fuel to useable thrust
- Includes installation effects such as
  - bleed air for cabin, electric generator, etc..
  - Inlet effects can be included (organizational dependent)
- Typical numbers are in range of 0.3 to 0.9. Can be up to 1.5
- Terminology varies with time units used, and it is not all consistent.
  - TSFC uses hours
  - "c" is often used for TSFC
  - Another term used is $$c_t \equiv \frac{\text{lb of fuel burned}}{\text{(lb of thrust delivered)(sec)}}$$

Breguet Range Equation

- Change in aircraft weight = fuel burned

$$dW = -c_t T dt \quad c_t = TSPC/3600 \quad T \equiv thrust$$

- Solve for dt and multiply by V_∞ to get ds

$$ds = V_\infty dt = -\frac{V_\infty dW}{c_t T} = -\frac{V_\infty W}{c_t T}\frac{dW}{W} = -\frac{V_\infty}{c_t}\frac{L}{D}\frac{dW}{W}$$

- Set L/D, c_t, V_∞ constant and integrate

$$R = \frac{3600}{TSFC} V_\infty \frac{L}{D} \ln \frac{W_{TO}}{W_{empty}}$$

Insights from Breguet Range Equation

$$R = \frac{3600}{TSFC} V_\infty \frac{L}{D} \ln\left(\frac{W_{TO}}{W_{empty}}\right)$$

$\frac{3600}{TSFC}$ represents propulsion effects. Lower TSFC is better

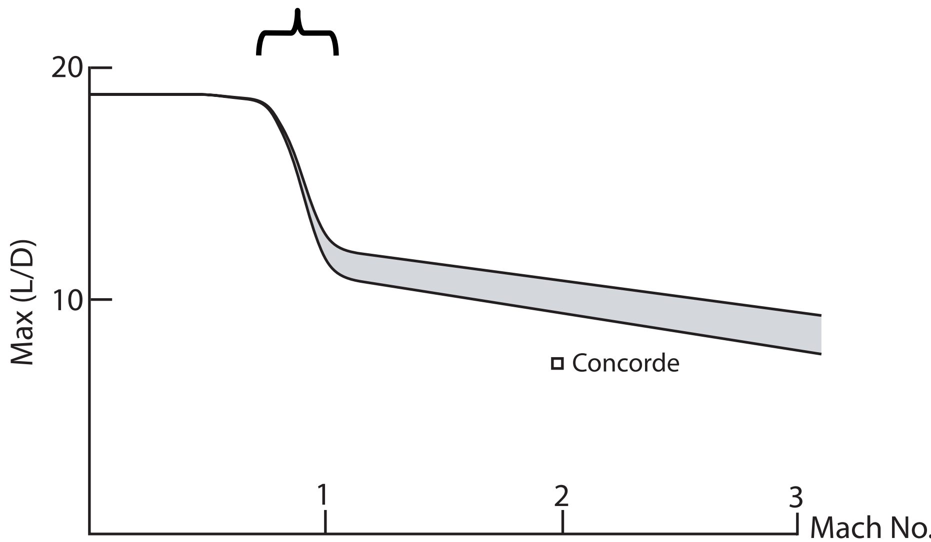
$V_\infty \frac{L}{D}$ represents aerodynamic effect. L/D is aerodynamic efficiency

$V_\infty \frac{L}{D} = a_\infty M_\infty \frac{L}{D}$. a_∞ is constant above 36,000 ft. $M_\infty \frac{L}{D}$ important

$\ln\left(\frac{W_{TO}}{W_{empty}}\right)$ represents aircraft weight/structures effect on range

Optimized L/D - Transport A/C

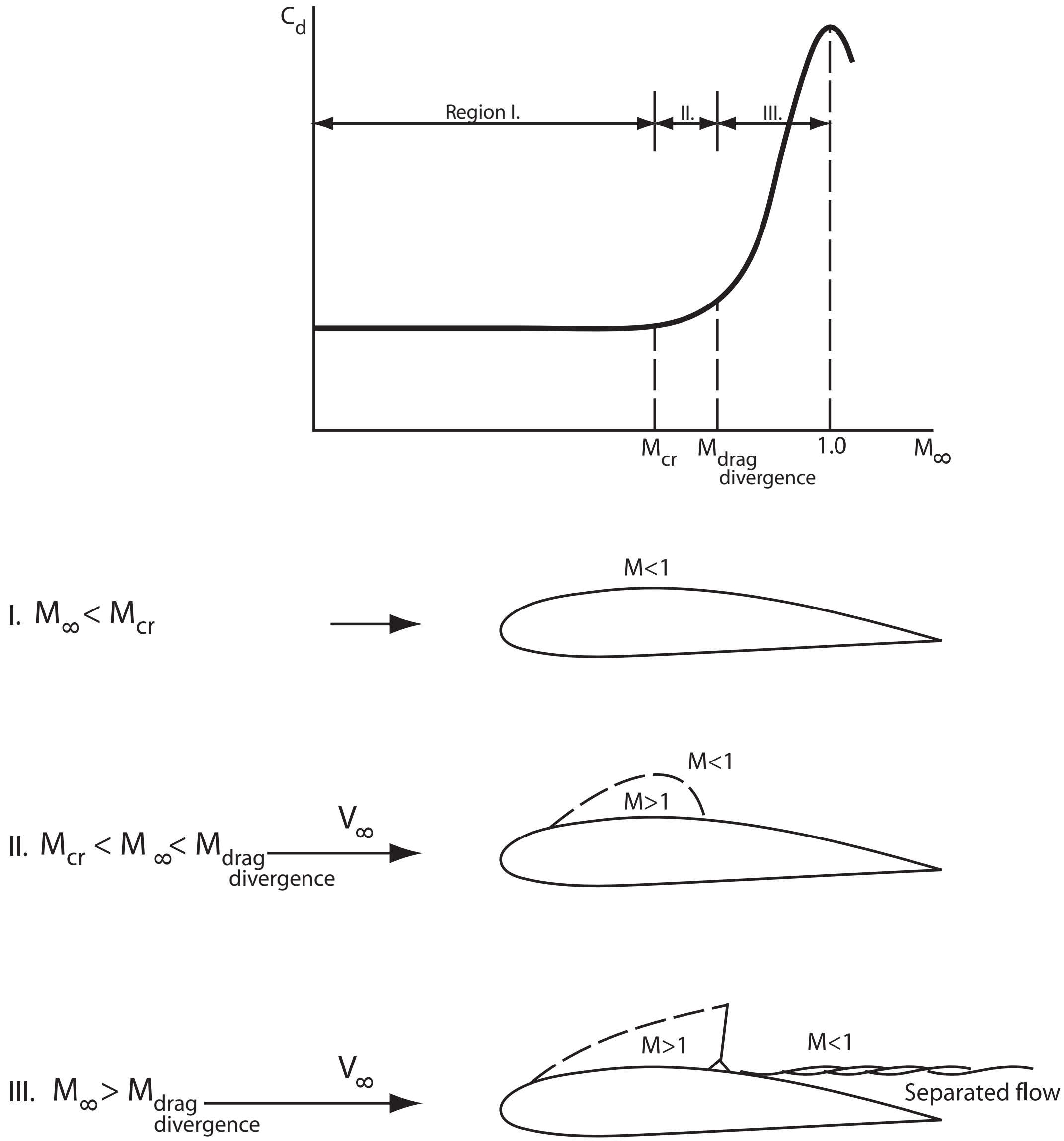
"Sweet spot" is in transonic range.



Losses due to shock waves

Ref: Shevell

Transonic Effects on Airfoil C_d, C_l

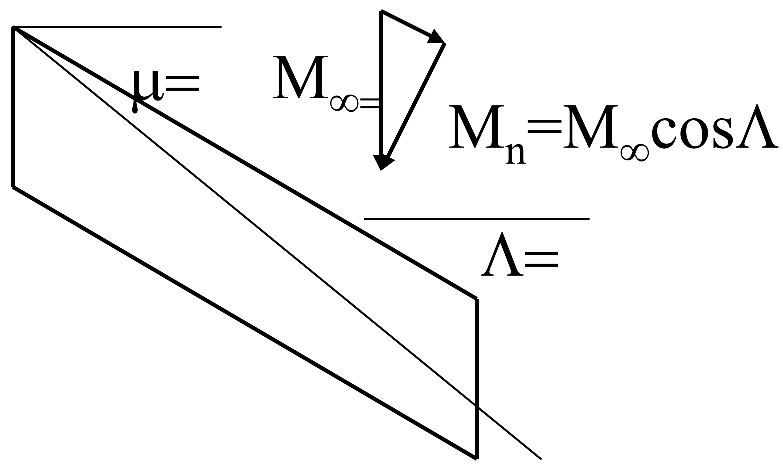


Strategies for Mitigating Transonic Effects

- Wing sweep
  - Developed by Germans. Discovered after WWII by Boeing
  - Incorporated in B-52
- Area Ruling, aka "coke bottling"
  - Developed by Dick Whitcomb at NASA Langley in 1954
    - Kucheman in Germany and Hayes at North American contributors
  - Incorporated in F-102
- Supercritical airfoils
  - Developed by Dick Whitcomb at NASA Langley in 1965
    - Percey at RAE had some early contributions
  - Incorporated in modern military and commercial aircraft

Basic Sweep Concept

- Consider Mach Number normal to leading edge



$\sin \mu = 1/ M_\infty$

μ = Mach angle, the direction disturbances travel in supersonic flow

- For subsonic freestreams, $M_n < M_\infty$ - Lower effective "freestream" Mach number delays onset of transonic drag rise.
- For supersonic freestreams
  - $M_n < 1$, $\Lambda > \mu$ = Subsonic leading edge
  - $M_n > 1$, $\Lambda < \mu$ = Supersonic leading edge
- Extensive analysis available, but this is gist of the concept

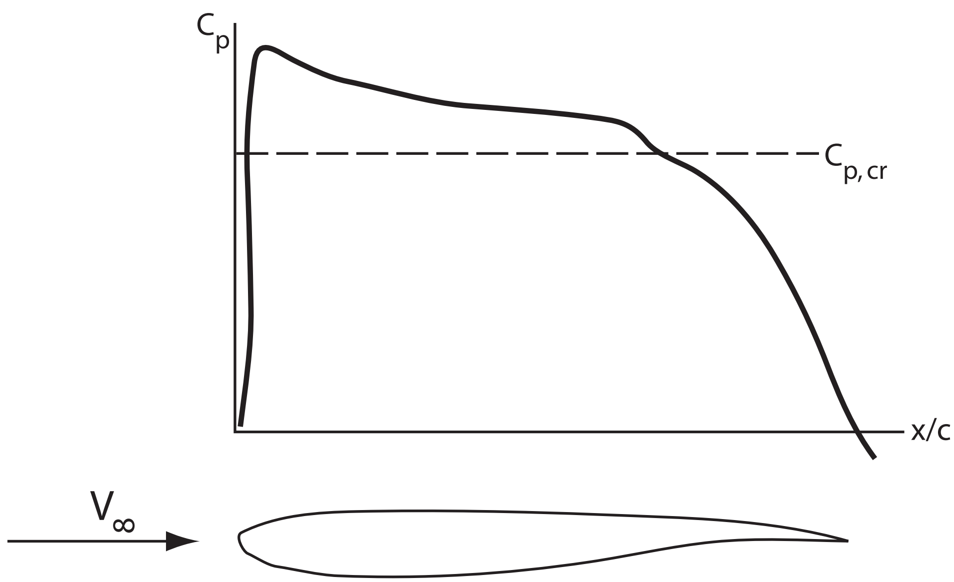
Wing Sweep Considerations $M_{\infty=} > 1$

- Subsonic leading edge
  - Can have rounded subsonic type wing section
    - Thicker section
    - Upper surface suction
    - More lift and less drag
- Supersonic leading edge
  - Need supersonic type wing section
    - Thin section
    - Sharp leading edge

Competing Needs

- Subsonic Mach number
  - High Aspect Ratio for low induced drag
- Supersonic Mach number
  - Want high sweep for subsonic leading edge
- Possible solutions
  - Variable sweep wing - B-1
  - Double delta - US SST
  - Blended - Concorde
  - Optimize for supersonic - B-58

Supercritical Airfoil



Supercritical airfoil shape keeps upper surface velocity from getting too large.

Uses aft camber to generate lift.

Gives nose down pitching moment.

Today's Performance Topics

- Turning analysis
  - Critical for high performance military a/c. Applicable to all.
  - Horizontal, pull-up, pull-down, pull-over, vertical
  - Universal M-ω-turn rate chart , V-n diagram
- Energy analysis
  - Critical for high performance military a/c. Applicable to all.
  - Specific energy, specific excess power
  - M-h diagram, min time to climb
- Operating envelope
- Back up charts for fighter aircraft
  - M-ω-diagram - "Doghouse" chart
  - Maneuver limits and some example
  - Extensive notes from Lockheed available. Ask me.

Horizontal Turn

$W = L\ cos\phi$, $\phi \equiv$ bank angle

Level turn, no loss of altitude

$F_r = (L^2 - W^2)^{1/2} = W(n^2-1)^{1/2}$

Where $n \equiv L/W = 1/\ cos\phi$ is the <u>load factor</u> measured in "g's".

But $F_r = (W/g)(V^2_\infty/R)$

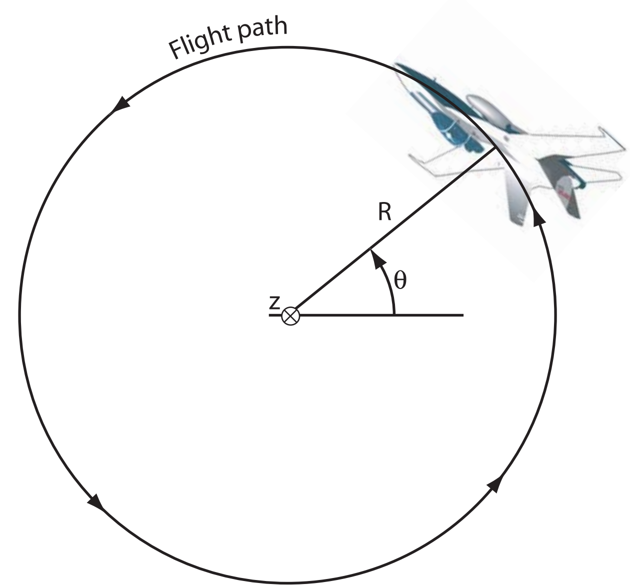
So radius of turn is

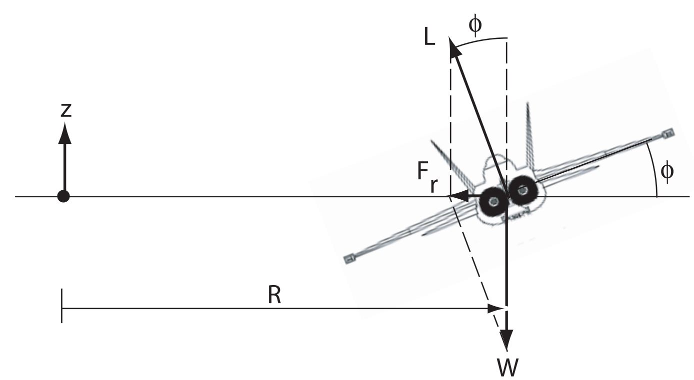
$$R = V^2_\infty/g(n^2-1)^{1/2}$$

And turn rate $\omega \equiv V_\infty/R$ is

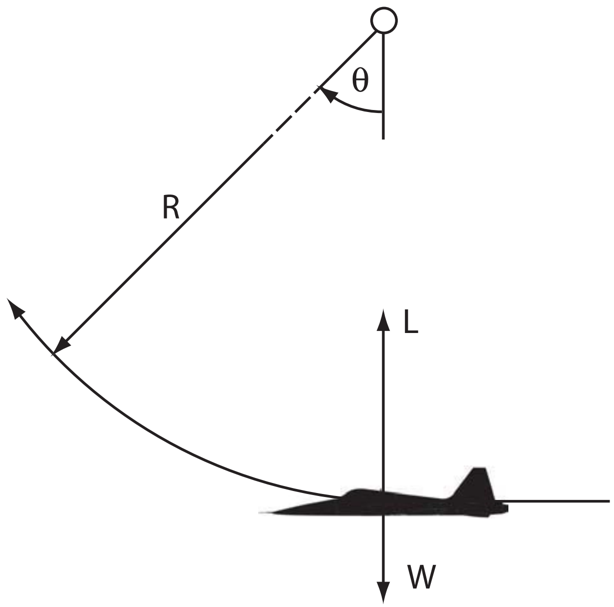
$$\omega = g(n^2-1)^{1/2} / V_\infty$$

Want high load factor, low velocity





Pull Up



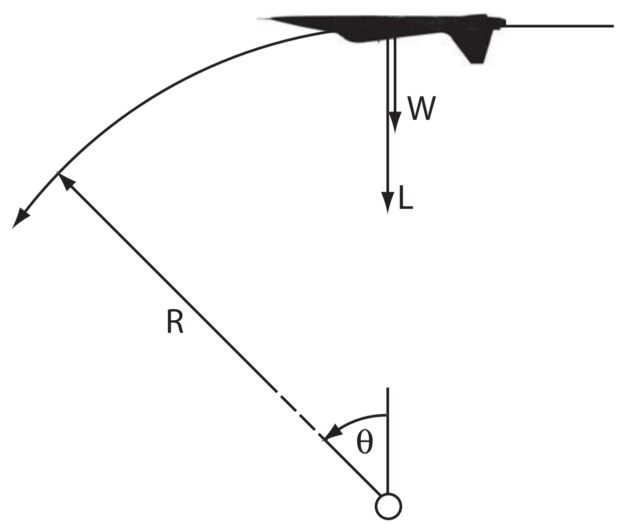
$$F_r = (L-W) = W(n-1)$$

$$= (W/g)(V^2_\infty/R)$$

$$R = V^2_\infty/g(n-1)$$

$$\omega = g(n-1)/ V_\infty$$

Pull Over



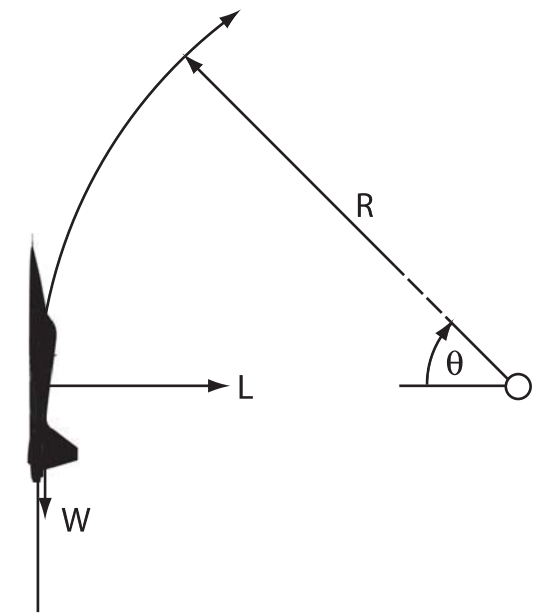
$$F_r = (L+W) = W(n+1)$$

$$= (W/g)(V^2_\infty/R)$$

$$R = V^2_\infty/g(n+1)$$

$$\omega = g(n+1)/ V_\infty$$

Vertical



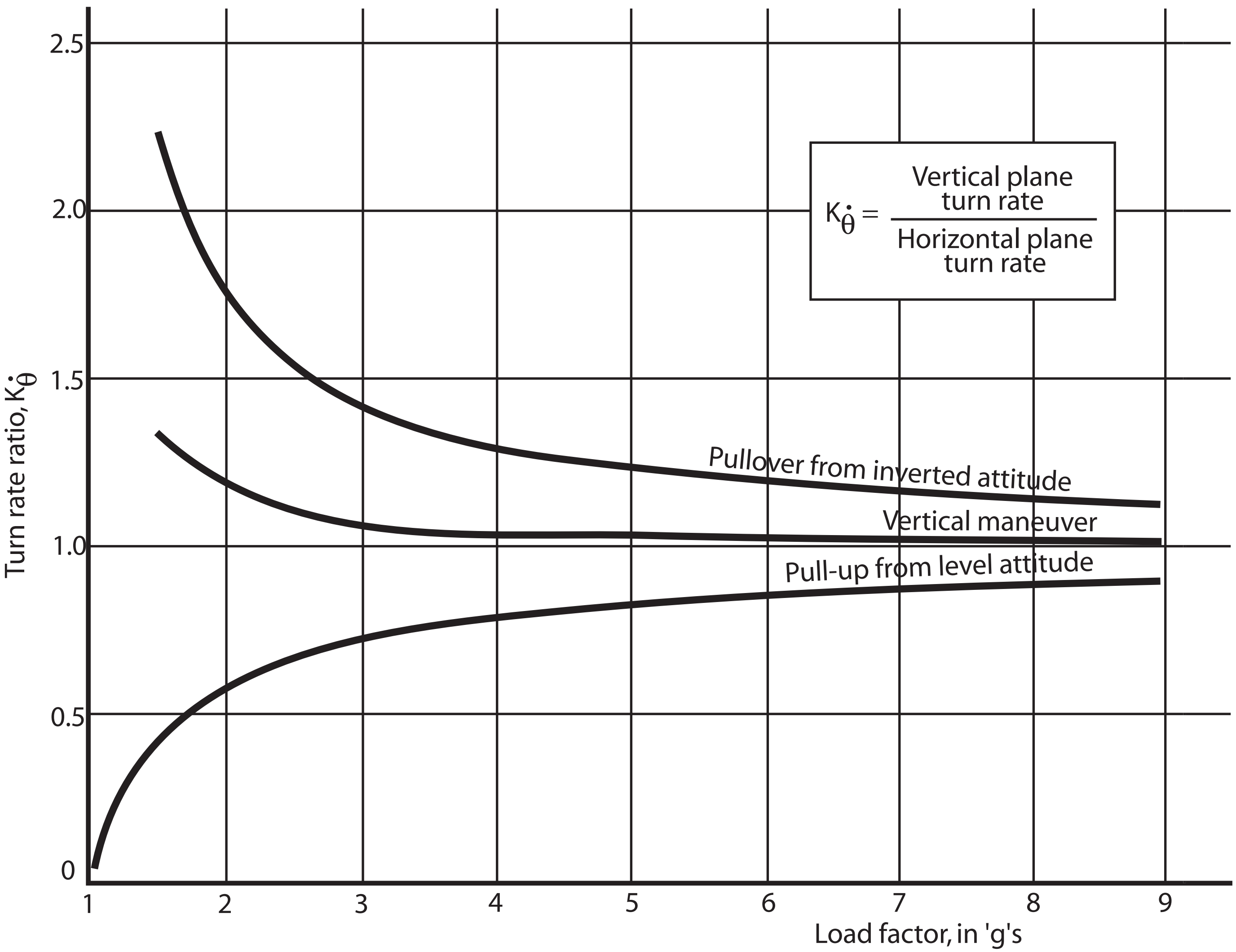
$$F_r = L = Wn$$

$$= (W/g)(V^2_\infty/R)$$

$$R = V^2_\infty/gn$$

$$\omega = gn/ V_\infty$$

Vertical Plan Turn Rates



Let $\dot{\theta} = \omega$

Pull Over

$K_{\omega} = (n+1)/(n^2-1)^{1/2}$

Vertical Maneuver

$K_{\omega} = n/(n^2-1)^{1/2}$

Pull Up

$K_{\omega} = (n-1)/(n^2-1)^{1/2}$

For large n, $K_{\omega} \to 1$ and for all maneuvers $\omega \cong gn/V_{\infty}$

Similarly for turn radius, for large n, $R \cong V^2_{\infty}/gn$.

For large ω and small R, want large n and small V_{∞}

$\omega \cong gn/ V_\infty = gn/a_\infty M_\infty$ so $\omega \sim 1/ M_\infty$ at const h (altitude) & n

Using $R \cong V^2_\infty/gn$, $\omega \cong V_\infty/R = a_\infty M_\infty/R$. So $\omega \sim M_\infty$ at const h & R

For high Mach numbers, the turn radius gets large

R_{min} and ω_{max}

Using $V_\infty = (2L/\rho_\infty S C_L)^{1/2} = (2nW/\rho_\infty S C_L)^{1/2}$

$R \cong V^2_\infty/gn$ becomes $R = 2(W/S)/ g\rho_\infty C_L$

W/S = wing loading, an important performance parameter

And using $n = L/W = \rho_\infty V^2_\infty S C_L/2W$

$\omega \cong gn/ V_\infty = g\, \rho_\infty V_\infty C_L/2(W/S)$

For each airplane, W/S set by range, payload, V_{max}.

Then, for a given airplane

$R_{min} = 2(W/S)/ g\rho_\infty C_{L,max}$

$\omega_{max} = g\, \rho_\infty V_\infty C_{L,max} /2(W/S)$

Higher $C_{L,max}$ gives superior turning performance.

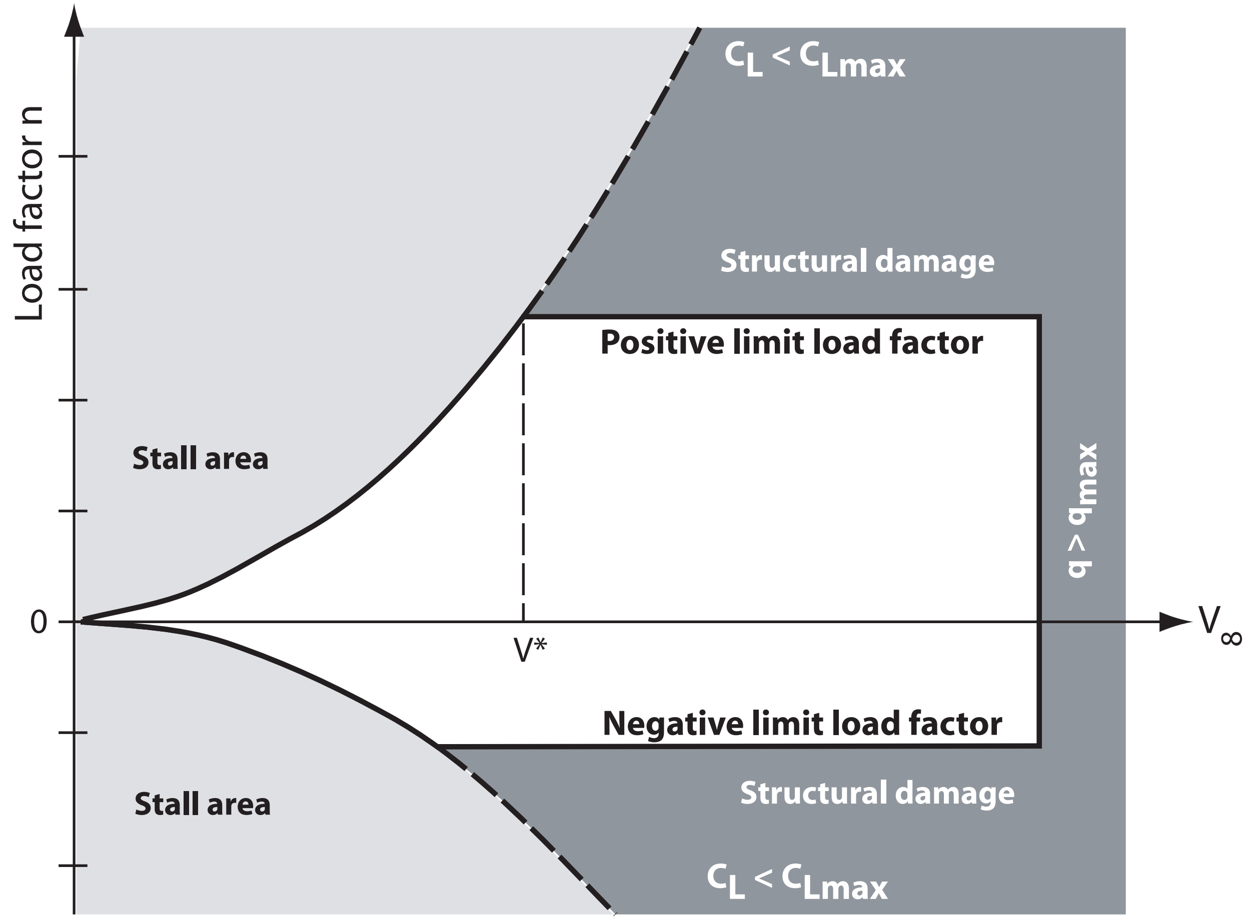
But does $n_{CL,max} = \rho_\infty V^2_\infty C_{L,max}/2(W/S)$ exceed structural limits?

V-n diagram

$$V^* \quad \sqrt{\frac{2n_{\max}}{\rho_\infty C_{L,\max}}\frac{W}{S}}$$

Highest possible ω=

Lowest possible R



Each airplane has a V-n diagram.

Source: Anderson

Summary on Turning

- Want large structural load factor n
- Want large $C_{L,MAX}$
- Want small V_∞
- Shortest turn radius, maximum turn rate is "Corner Velocity"

- Question, does the aircraft have the power to execute these maneuvers?

Specific Energy and Excess Power

Total aircraft energy = PE + KE

$E_{tot} = mgh + mV^2/2$

Specific energy = (PE + KE)/W

$H_e = h + V^2/2g$ "energy height"

Excess Power = (T-D)V

Specific excess power*

$= (TV-DV)/W$

$= dH_e/dt$

$P_s = dh/dt + V/g\ dV/dt$

P_s may be used to change altitude, or accelerate, or both

* Called specific power in Lockheed Martin notes.

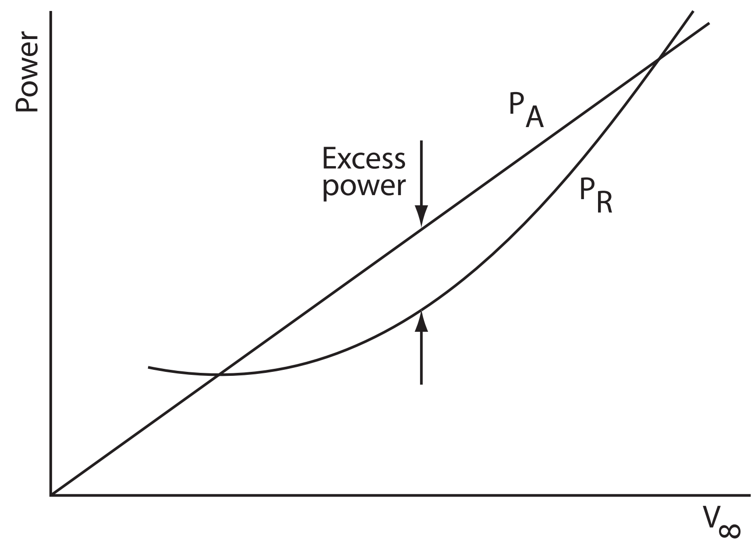
Excess Power

Power Required

$$P_R = DV_\infty = q_\infty S(C_{D,0} + C^2_L/\pi AR e)V_\infty$$

$$= q_\infty S C_{D,0} V_\infty + q_\infty S V_\infty C^2_L/\pi AR e$$

$$= \underbrace{\rho_\infty S C_{D,0} V^3_\infty/2}_{\text{Parasite power required}} + \underbrace{2n^2W^2/\rho_\infty S V_\infty \pi AR e}_{\text{Induced power required}}$$



Power Available

$P_A = TV_\infty$ and Thrust is approximately constant with velocity, but varies linearly with density.

Excess power depends upon velocity, altitude and load factor

Altitude Effects on Excess Power

$P_R = DV_\infty = (nW/L)\ DV_\infty$

$= nWV_\infty\ C_D/C_L$

From $L = \rho_\infty SV^2_\infty\ C_L/2 = nW$, get

$V_\infty = (2nW/\ \rho_\infty SC_L)^{1/2}$

Substitute in P_R to get

$P_R = (2n^3W^3C^2_D/\ \rho_\infty SC^3_L)^{1/2}$

So can scale between sea level "0" and altitude "alt" assuming C_D, C_L const.

$V_{alt} = V_0(\rho_0/\rho_{alt})^{1/2}$, $P_{R,alt} = P_{R,0}(\rho_0/\rho_{alt})^{1/2}$

Thrust scales with density, so

$P_{A,alt} = P_{A,0}(\rho_{alt}/\rho_0)$

Summary of Power Characteristics

- H_e = specific energy represents "state" of aircraft. Units are in feet.
  - Curves are universal
- $P_s = (T/W-D/W)V$ = specific excess power
  - Represents ability of aircraft to change energy state.
  - Curves depend upon aircraft (thrust and drag)
  - Maybe used to climb and/or accelerate
  - Function of altitude
  - Function of load factor
- "Military pilots fly with P_s diagrams in the cockpit", Anderson

A/C Performance Summary

Factor	Commercial Transport	Military Transport	Fighter	General Aviation
Take-off	Liebeck			
	h_{obs} = 35'	h_{obs} = 50'	h_{obs} = 50'	h_{obs} = 50'
Landing	Liebeck			
	$V_{app} = 1.3\ V_{stall}$	$V_{app} = 1.2\ V_{stall}$	$V_{app} = 1.2\ V_{stall}$	$V_{app} = 1.3\ V_{stall}$
Climb	Liebeck			
Level Flight	Liebeck			
Range	Breguet Range		Radius of action*. Uses refueling	Breguet for prop
Endurance, Loiter	E (hrs) = R (miles)/V(mph), where R = Breguet Range			
Turning, Maneuver	Emergency handling		Major performance factor	Emergency handling
Supersonic Dash	N/A	N/A	Important	N/A
Service Ceiling	100 fpm climb			

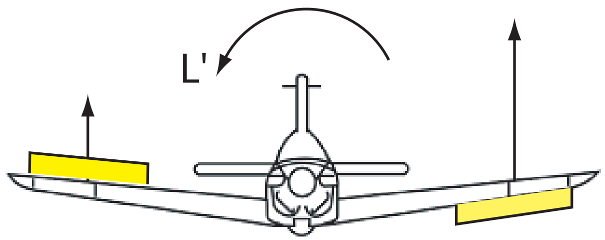
(Shaded cells:) Lectures 6 and 7 for commercial and military transport

* Radius of action comprised of outbound leg, on target leg, and return.

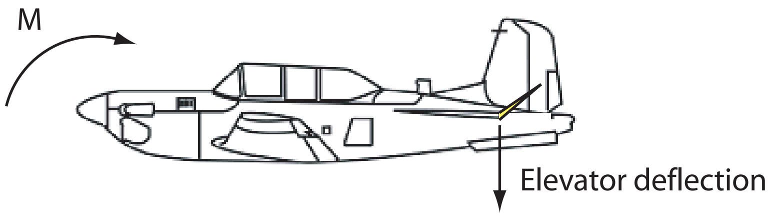
Stability and Control

- Performance topics deal with forces and translational motion needed to fulfill the aircraft mission
- Stability and control topics deal with moments and rotational motion needed for the aircraft to remain controllable.

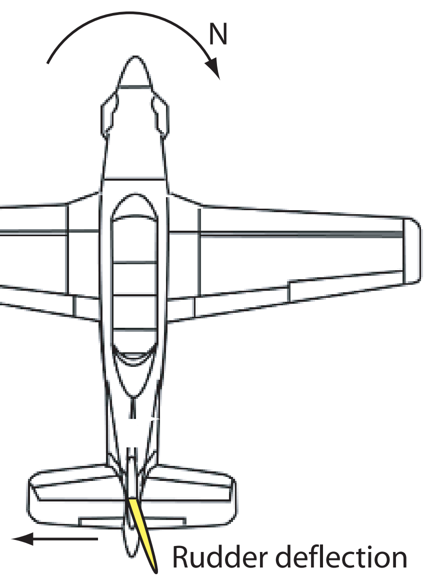
S&C Definitions



- L' - rolling moment
- Lateral motion/stability



- M - pitching moment
- Longitudinal motion/control



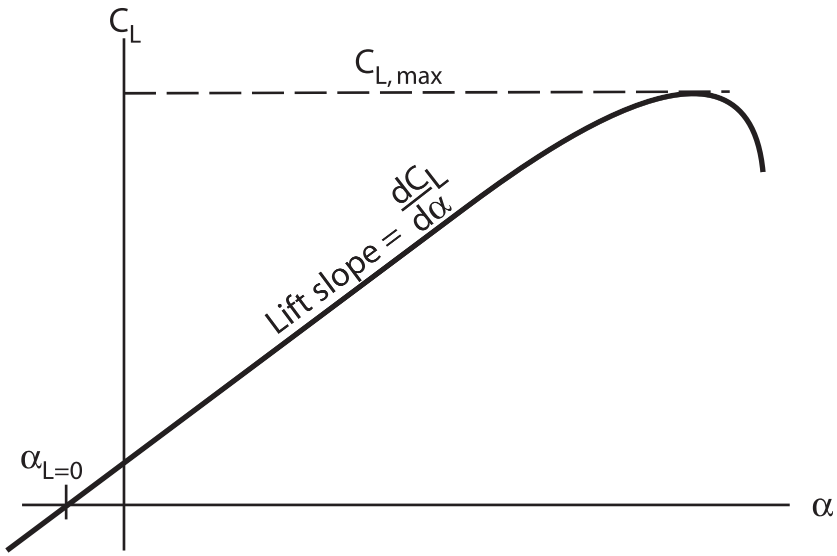
- N - rolling moment
- Directional motion/control

Moment coefficient: $C_M = \frac{M}{q_\infty S c}$

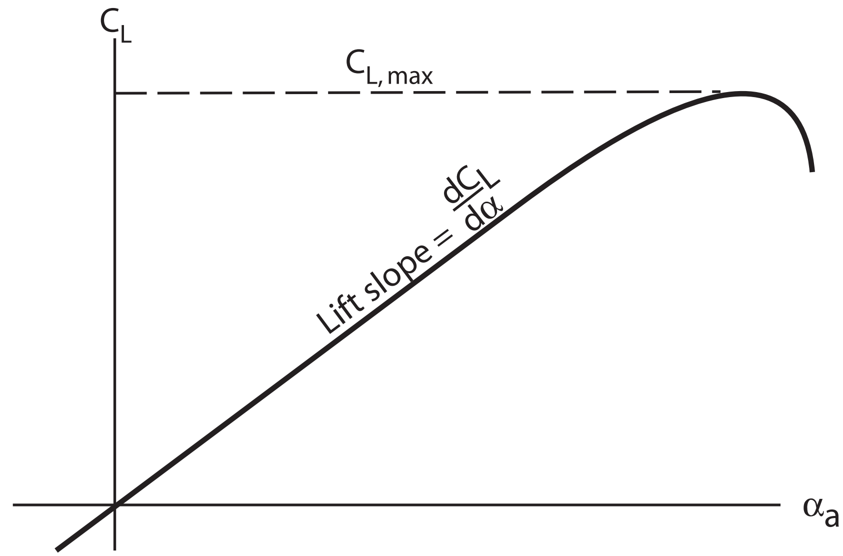
Aircraft Moments

- Aerodynamic center (ac): forces and moments can be completely specified by the lift and drag acting through the ac plus a moment about the ac
  - $C_{M,ac}$ is the aircraft pitching moment at L = 0 around any point
- Contributions to pitching moment about cg, $C_{M,cg}$ come from
  - Lift and $C_{M,ac}$
  - Thrust and drag - will neglect due to small vertical separation from cg
  - Lift on tail
- Airplane is "trimmed" when $C_{M,cg} = 0$

Absolute Angle of Attack



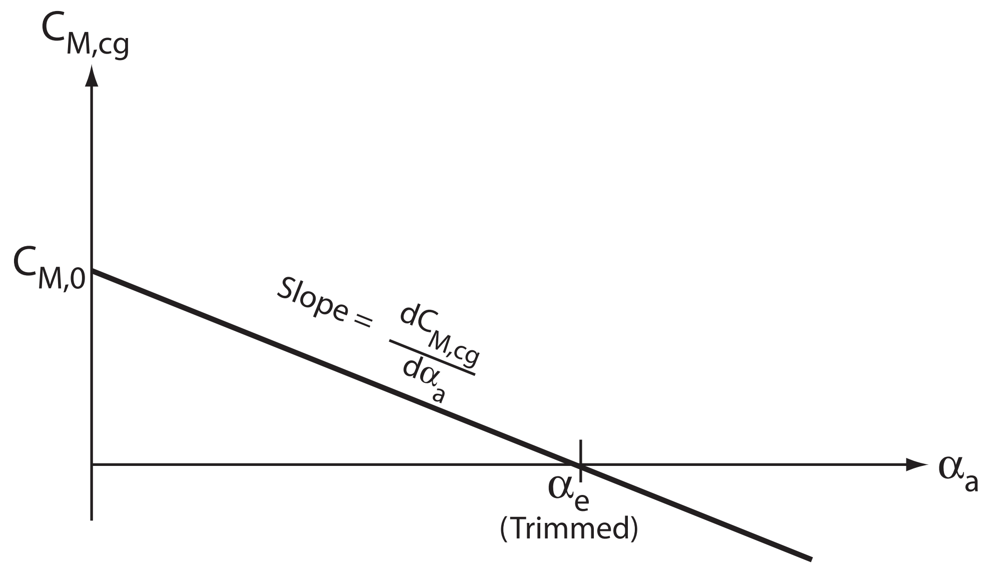
Lift coefficient vs geometric angle of attack, α



Lift coefficient vs absolute angle of attack, α_a

- Stability and control analysis simplified by using the absolute angle of attack which is 0 at $C_L = 0$.
- $\alpha_a = \alpha - \alpha_{L=0}$

Criteria for Longitudinal Static Stability



$C_{M,0}$ must be positive

$\frac{\partial C_{M,cg}}{\partial \alpha_a}$ must be negative

Implied that α_e is within flight range of angle of attack for the airplane, i.e. aircraft can be trimmed

Moment Around cg

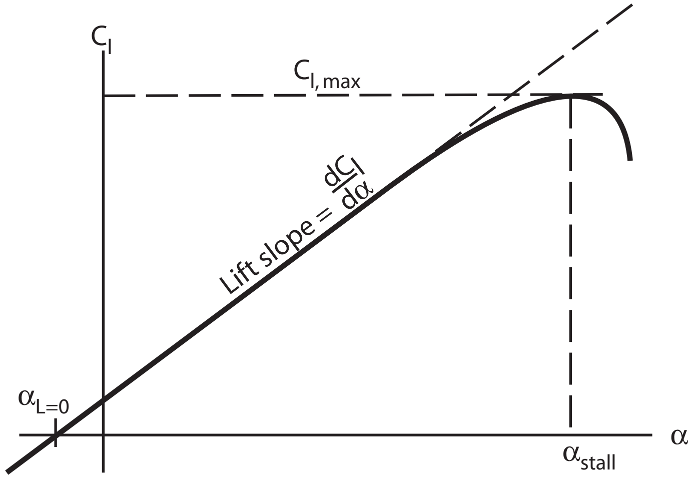
$$M_{cg} = M_{ac_{wb}} + L_{wb}(hc - h_{ac}c) - l_t L_t$$

Divide by $q_\infty Sc$ and note that $C_{L,t} = \dfrac{L_t}{q_\infty S_t}$

$$C_{M,cg} = C_{M,ac_{wb}} + C_{L_{wb}}(h - h_{ac}) - \frac{l_t S_t}{cS} C_{L,t}$$, or

$$\boxed{C_{M,cg} = C_{M,ac_{wb}} + C_{L_{wb}}(h - h_{ac}) - V_H C_{L,t}}$$, where $V_H = \dfrac{l_t S_t}{cS}$

$$C_{M,cg} = C_{M,ac_{wb}} + C_{L_{wb}}(h - h_{ac}) - V_H C_{L,t}$$



$$C_{L_{wb}} = \frac{dC_{L_{wb}}}{d\alpha}\alpha_{a,wb} = a_{wb}\alpha_{a,wb}$$

$$C_{l,t} = a_t\alpha_t = a_t(\alpha_{wb} - i_t - \varepsilon)$$

where ε is the downwash at the tail due to the lift on the wing

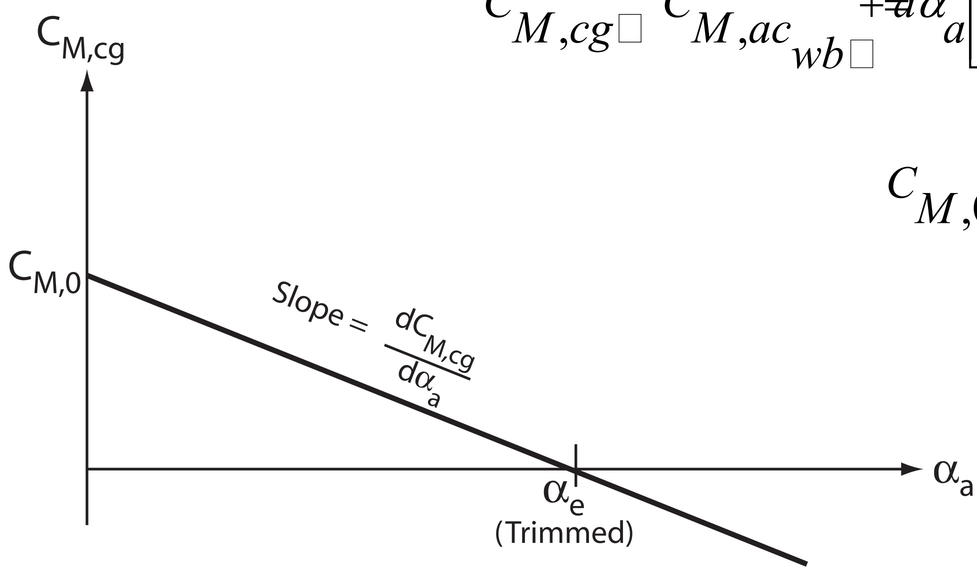
$$\varepsilon = \varepsilon_0 + \left(\frac{\partial\varepsilon}{\partial\alpha}\right)\alpha_{a,wb}$$

$$C_{L,t} = a_t\alpha_{a,wb}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) - a_t(i_t + \varepsilon_0)$$

At this point, the convention is drop the *wb* on a_{wb}

$$C_{M,cg} = C_{M,ac_{wb}} + a\alpha_a\left[(h - h_{ac}) - V_H\frac{a_t}{a}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right] + V_H a_t(i_t + \varepsilon_0)$$

Eqs for Longitudinal Static Stability



$$C_{M,cg} = C_{M,ac_{wb}} + a\alpha_a\left[(h - h_{ac}) - V_H \frac{a_t}{a}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right] + V_H a_t\left(i_t + \varepsilon_0\right)$$

$$C_{M,0} = \left(C_{M,cg}\right)_{l=0} = C_{M,ac_{wb}} + V_H a_t\left(i_t + \varepsilon_0\right)$$

$$\frac{\partial C_{M,cg}}{\partial\alpha_a} = a\left[(h - h_{ac}) - V_H \frac{a_t}{a}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right]$$

- $C_{M,acwb} < 0$, $V_H > 0$, $\alpha_t > 0 \Rightarrow i_t > 0$ for $C_{M,0} > 0$
  - Tail must be angled down to generate negative lift to trim airplane
- Major effect of cg location (h) and tail parameter $V_H = (lS)_t/(cs)$ in determining longitudinal static stability

Neutral Point and Static Margin

$$\frac{\partial C_{M,cg}}{\partial \alpha} = a\left[(h - h_{ac}) - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

- The slope of the moment curve will vary with *h*, the location of cg.
- If the slope is zero, the aircraft has neutral longitudinal static stability.
- Let this location be denote by $h_n = h_{ac} + V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$
- or $\frac{\partial C_{M,cg}}{\partial \alpha} = a\left(h - h_n\right) = -a\left(h_n - h\right) = -a \times$ static margin

- For a given airplane, the neutral point is at a fixed location.
- For longitudinal static stability, the position of the center of gravity must always be forward of the neutral point.
- The larger the static margin, the more stable the airplane

Longitudinal Static Stability

Aerodynamic center location moves aft for supersonic flight

cg shifts with fuel burn, stores separation, configuration changes

- "Balancing" is a significant design requirement
- Amount of static stability affects handling qualities
- Fly-by-wire controls required for statically unstable aircraft

Today’s References

- **Lockheed Martin Notes on “Fighter Performance”**
- **John Anderson Jr. , *Introduction to Flight*, McGraw-Hill, 3rd ed, 1989, Particularly Chapter 6 and 7**
- **Shevell, Richard S., “Fundamentals of Flight”, Prentice Hall, 2nd Edition, 1989**
- **Bertin, John J. and Smith, Michael L., Aerodynamics for Engineers, Prentice Hall, 3rd edition, 1998**
- **Daniel Raymer, *Aircraft Design: A Conceptual Approach*, AIAA Education Series, 3rd edition, 1999, Particularly Chapter 17**
  - **Note: There are extensive cost and weight estimation relationships in Raymer for military aircraft.**