Introduction to Optics part I

Overview Lecture

Space Systems Engineering

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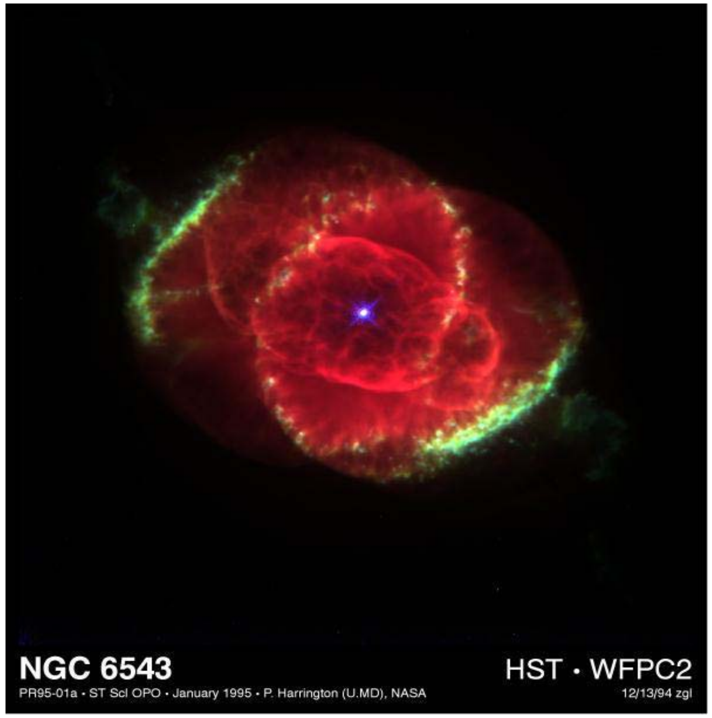
Outline

Goal: Give necessary optics background to tackle a space mission, which includes an optical payload

- Light
- Interaction of Light w/ environment
- Optical design fundamentals
- Optical performance considerations
- Telescope types and CCD design
- Interferometer types
- Sparse aperture array
- Beam combining and Control

Examples - Motivation

Spaceborne Astronomy

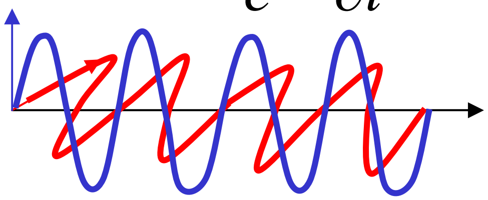


Planetary nebulae NGC 6543
September 18, 1994
Hubble Space Telescope

Properties of Light

Wave Nature ← **Duality** → Particle Nature

$$\nabla^2 E - \frac{\varepsilon\mu}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$



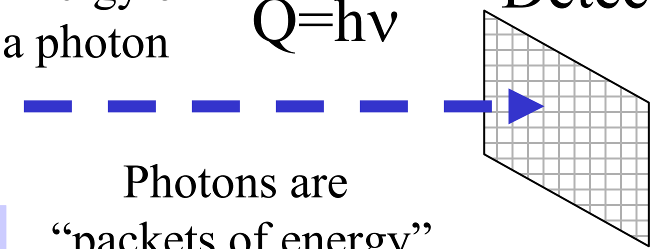
E: Electric field vector
H: Magnetic field vector

Solution:

$$E = Ae^{i(kr - \omega t + \phi)}$$

Energy of a photon $Q = h\nu$

Detector



Photons are "packets of energy"

Poynting Vector: $S = \frac{c}{4\pi} E \times H$

Spectral Bands (wavelength λ):

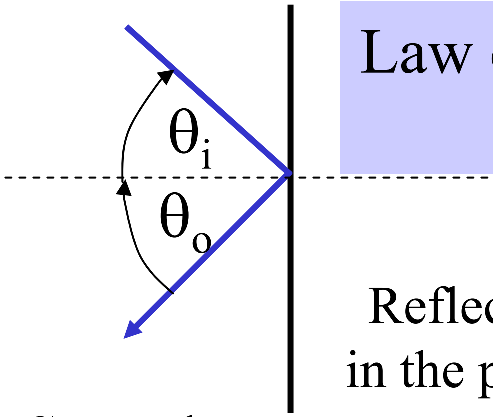
Ultraviolet (UV) 300 Å -300 nm
Visible Light 400 nm - 700 nm
Near IR (NIR) 700 nm - 2.5 μm

Wavelength: $\lambda = \nu \frac{2\pi}{\omega} = \nu T$

Wave Number: $k = \frac{2\pi}{\lambda}$

Reflection-Mirrors

Mirrors (Reflective Devices) and Lenses (Refractive Devices) are both "Apertures" and are similar to each other.



Specular Reflection

Law of reflection:

$\theta_i = \theta_o$

Reflected wave is also in the plane of incidence

Mirror Geometry given as a conic section rot surface:

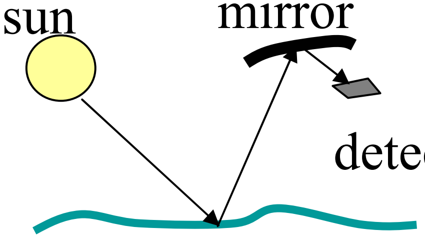
$$z(\rho) = \frac{1}{k+1}\left(r - \sqrt{r^2 - (k+1)\rho^2}\right)$$

Circle: $k=0$ Ellipse $-1<k<0$

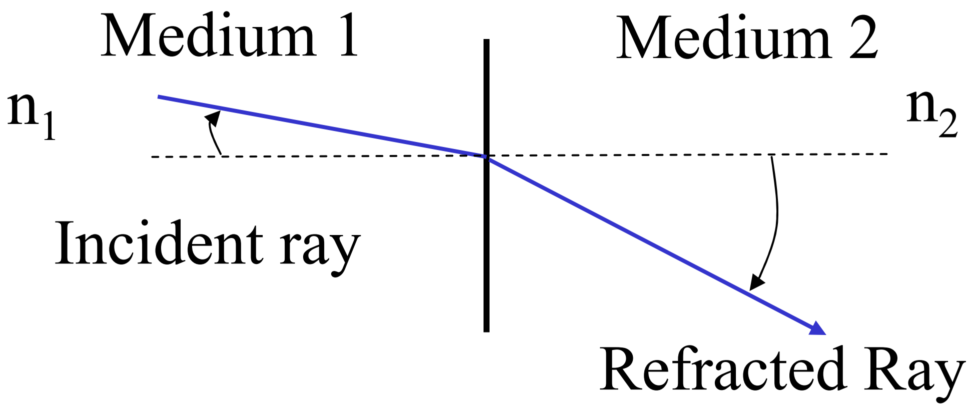
Parabola: $k=-1$ Hyperbola: $k<-1$

Detectors resolve Images produced by (solar) energy reflected from a target scene* in Visual and NIR.

*rather than self-emissions



Transmission-Refraction



Recall Snell's Law

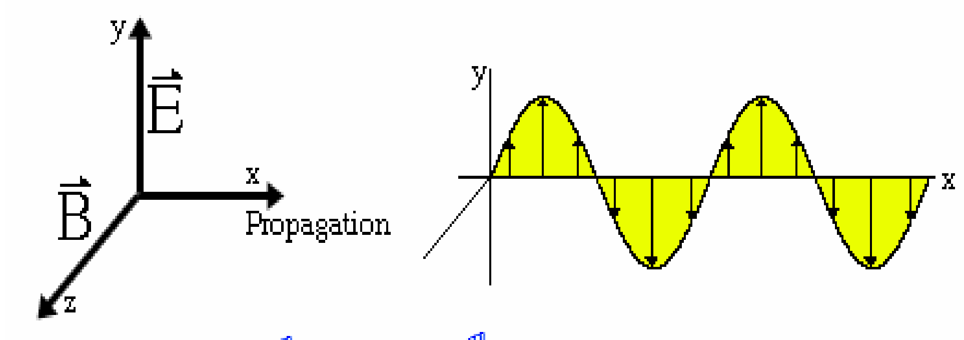
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Light Intensity $S = \frac{c}{4\pi}\sqrt{\varepsilon}E^2$

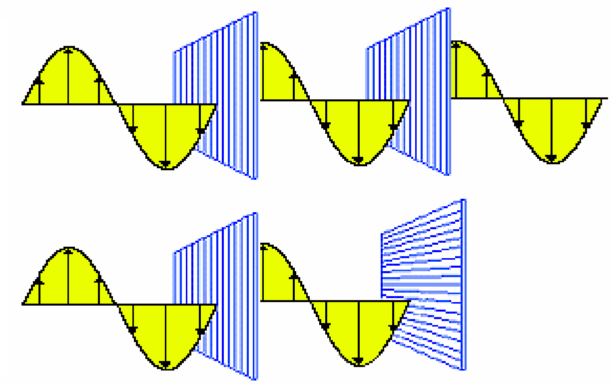
Dispersion if index of refraction is wavelength dependent $n(\lambda)$

Refractive devices not popular in space imaging , since we need different lenses for UV, visual and IR.

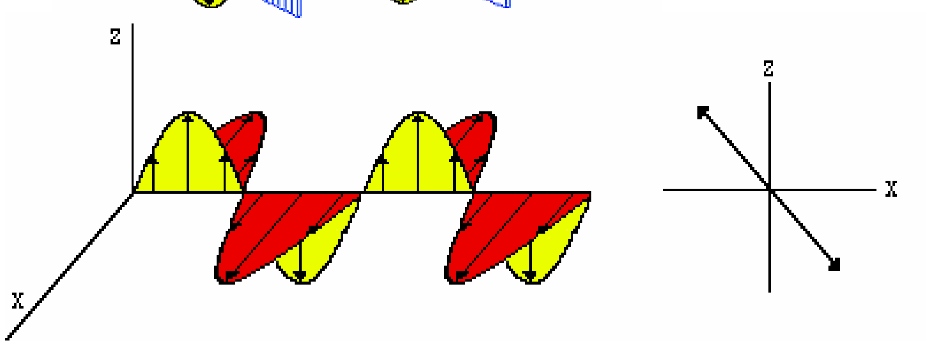
Polarization



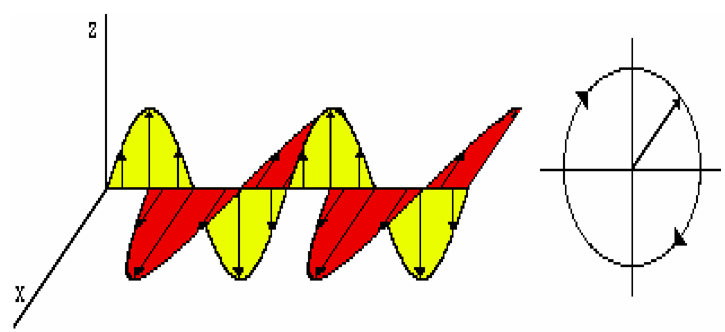
Light can be represented as a transverse electromagnetic wave made up of mutually perpendicular, fluctuating electric and magnetic fields.



Ordinary white light is made up of waves that fluctuate at all possible angles. Light is considered to be "linearly polarized" when it contains waves that only fluctuate in one specific plan (Polarizers are shown)



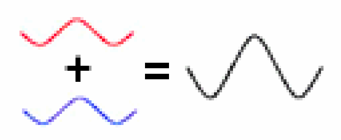
In-phase=> 45 degrees linearly polarized



90 degree out of phase->circular

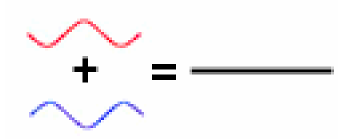
Interference

Interference: Interaction of *two or more light waves* yielding a resultant irradiance that deviates from the sum of the component irradiances



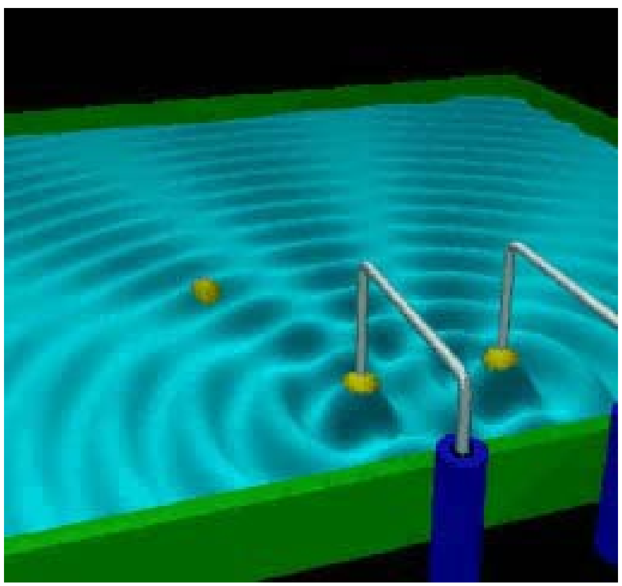
If the high part of one wave (its crest) overlaps precisely with the high part of another wave, we get enhanced light.
Crest + Crest = Strong Light

$$(r_1 - r_2) = 2\pi m / k = m\lambda$$



If the high part of one wave overlaps precisely with the low part of another wave (its trough), they cancel each other out.
Crest + Trough = Darkness

$$(r_1 - r_2) = \pi m / k = \frac{1}{2} m\lambda$$



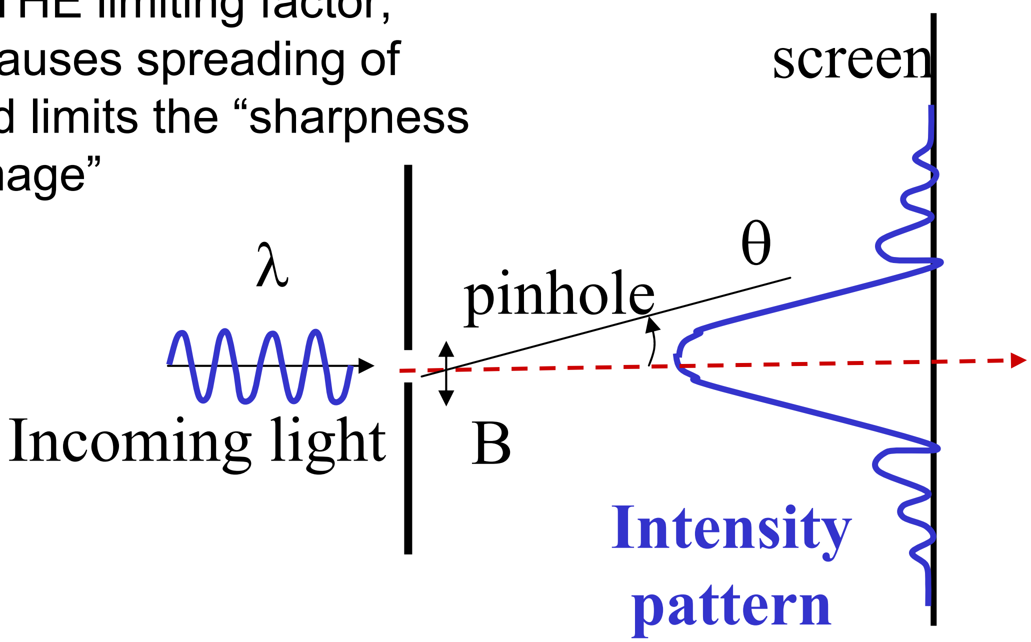
Conditions of Interference:

- need not be in phase with each source, but the initial phase difference remains constant (coherent)
- A stable fringe pattern must have nearly the same frequency. But,white light will produce less sharp, observable interference
- should not be orthogonally polarized to each other

Diffraction

Diffraction occurs at the edges of optical elements and field stops, this limits the Field-of-View (FOV).

This is THE limiting factor, which causes spreading of light and limits the “sharpness of an image”



$$I = |E|^2 = I_o \left(\frac{\sin u}{u} \right)^2$$

$$u = \frac{\pi}{\lambda} B \sin\theta$$

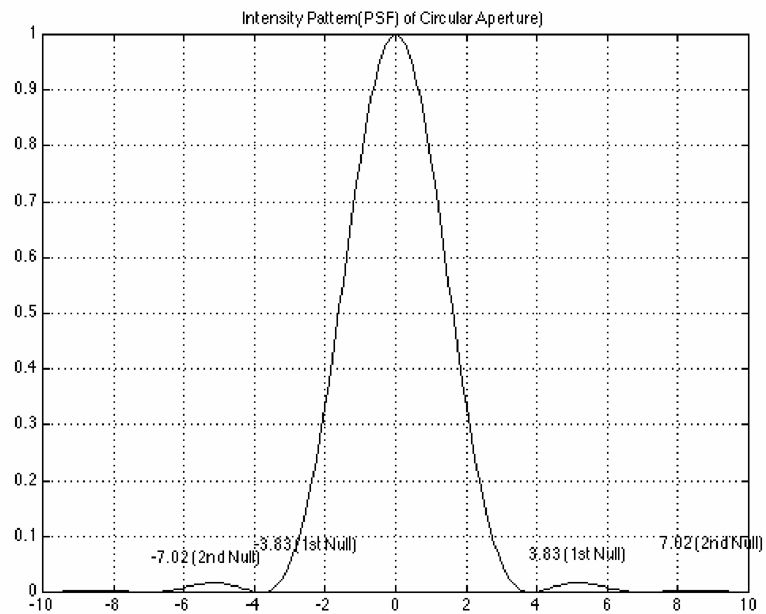
B - aperture size

θ - angle of boresight

Fraunhofer Diffraction Thoery (very distant object) is applied.

sine function is replaced by $\mathbf{J_1}$ for a circular aperture

Derivation of Angular Resolution



$$I = |E|^2 = I_o \left(\frac{2J_1(u)}{u} \right)^2$$

J_1:Bessel function of the first kind(order 1)

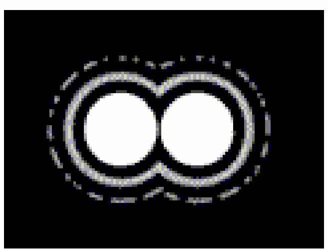
$$u = 3.83 = \frac{\pi}{\lambda} B \sin\theta = \frac{\pi}{\lambda} B\theta$$

$$\theta = \frac{3.83\lambda}{\pi B} = \frac{1.22\lambda}{B}$$

B - aperture size

θ - angle of boresight

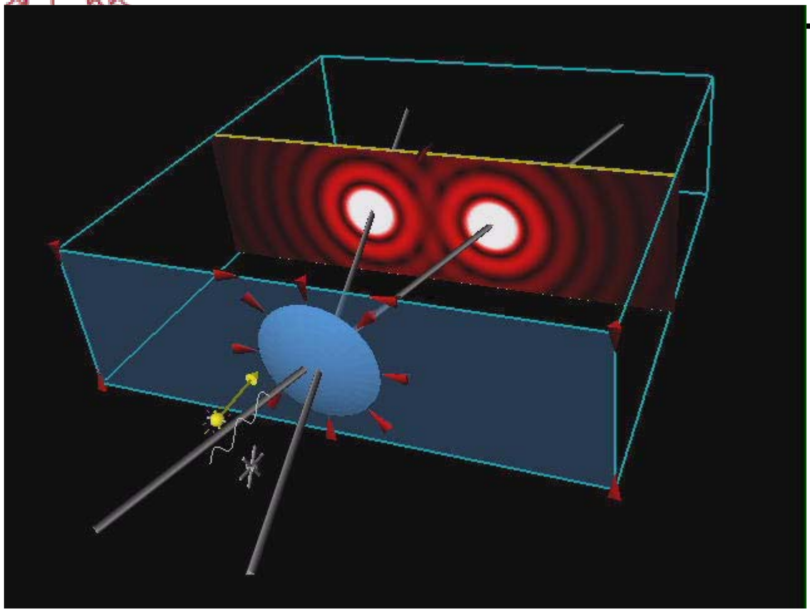
=> Rayleigh Criterion

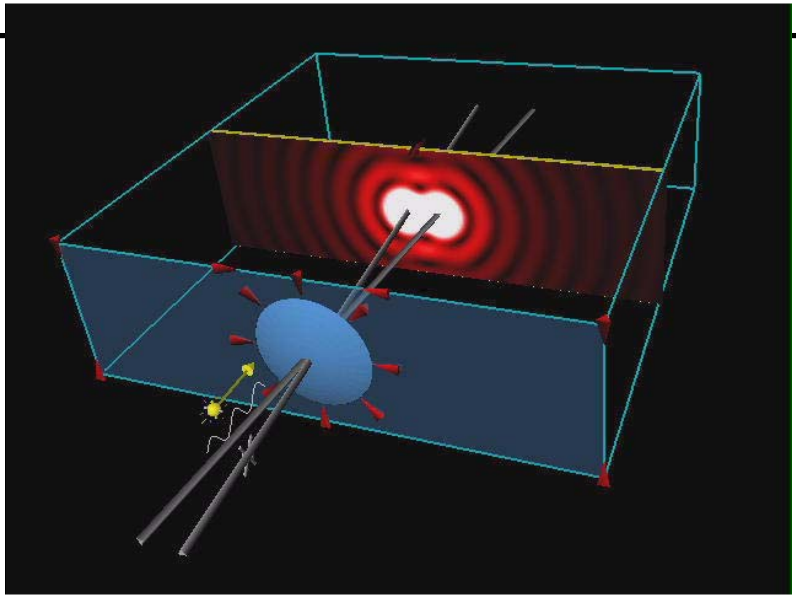


Angular Resolution(Resolving Power) :the telescope's ability to clearly separate, or resolve, two star points (i.e., two Airy discs)

Goal is to design optical system to be diffraction limited at the wavelength of interest.

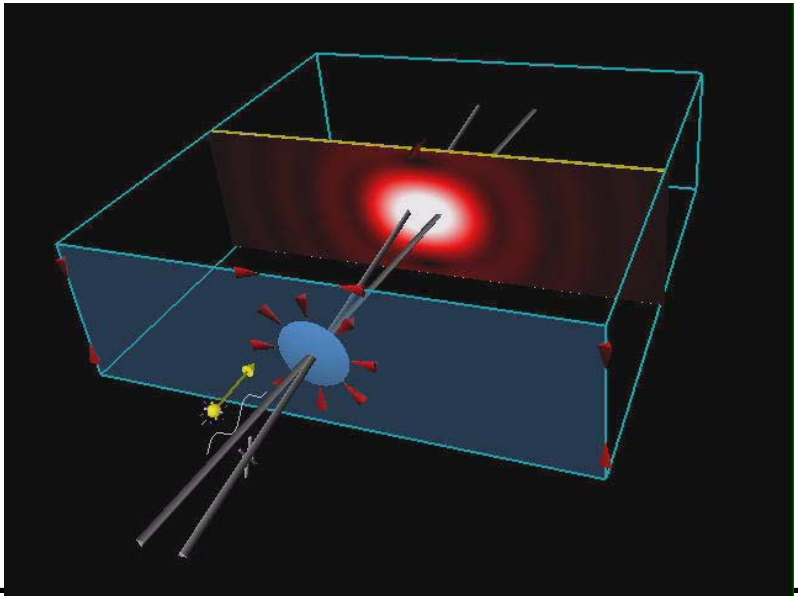
Angular Resolution Simulation





$$\theta = \frac{1.22\lambda}{B}$$

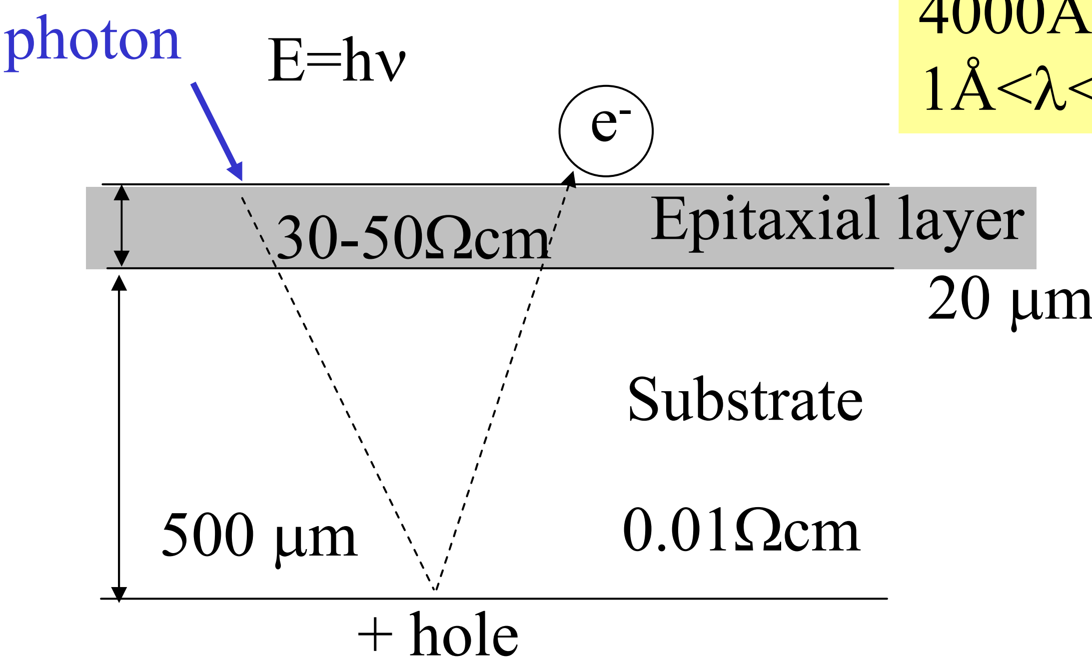
Effects of separation, diameter and wavelength on Resolving Power



Absorption

Electronic Detectors work by absorption, i.e. a photon is absorbed by a semiconductor surface and turned into a photo-electron -> photoelectric effect.

$4000Å < \lambda < 10{,}000Å$, $e^- = 1$
$1Å < \lambda < 1000Å$, $e^- = eV/3.65eV/e^-$



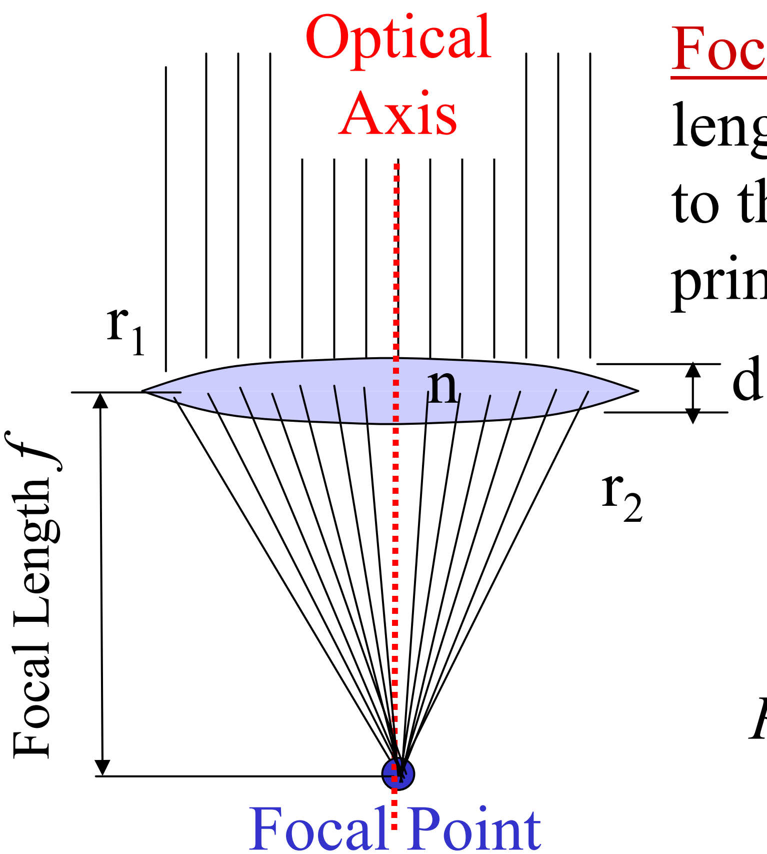
# of photoelectrons generated.

Absorption in an opaque non-silicon opaque material photons -> heat

E.g. Germanium is opaque in visible but transmissive in the band from 1.8-25 µm. Opaque surfaces absorb.

Optical Design Fundamentals (1)

Systems for gathering and transmitting RF (radio frequency) and optical signals are identical in theory. Hardware is different.



Focal length ***f*** determines overall length of optical train and is related to the radius of curvature (ROC) of the primary mirror/lens surface.

Power of a lens/mirror:

$$P = 1/f \qquad [\text{diopters}=\text{m}^{-1}]$$

Lensmakers Formula:

$$P = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{(n-1)^2}{n}\frac{d}{r_1 r_2}$$

In principle: Optical Mirror ~ RF Parabolic Dish Antenna

Optical Design Fundamentals (2)

Approach (I) for determining the focal length ***f***

Required Field of View (FOV)

Size of Image Plane [m]*

→

Plate Scale s

$s = f$

[m on focal plane/rad on sky]

E.g. "1cm on the focal plane equals 2 km on the ground"

Focal length f needed to record a scene of Radius *R on the ground*:

Important Equation !!!

$$\frac{f}{h} = \frac{r_d}{R} = m \text{ magnification}$$

f: focal length [m]

h: altitude [m]

r_d: radius detector array [m]

R: Target radius [m]

* can arrange several detectors (CCD's) in a matrix to obtain a larger image on the focal plane

Optical Design Fundamentals (3)

Approach (II) for determining the focal length *f*

Detect point targets at a fixed range:

There is a central bright ring containing 83.9% of the total energy passing through the aperture. Angular dimension of this ring is:

$$d_{AIRY} = 2.44\frac{\lambda}{D}$$

Required focal length ***f*** to give an image of diameter Γ for a point target:

$$f = \frac{\Gamma D}{2.44\lambda}$$

λ : wavelength of light
D: aperture diameter
Γ: image diameter of a point target

Diffraction spreads the light:

Telescope Key Variables

Most important: **f** - Focal length

$$F\# = \frac{f}{D} = \frac{1}{2NA}$$

Infinity F-number* , e.g. F# or f/

* a.k.a. F-stop: synonyms: f/, F, F No., F#

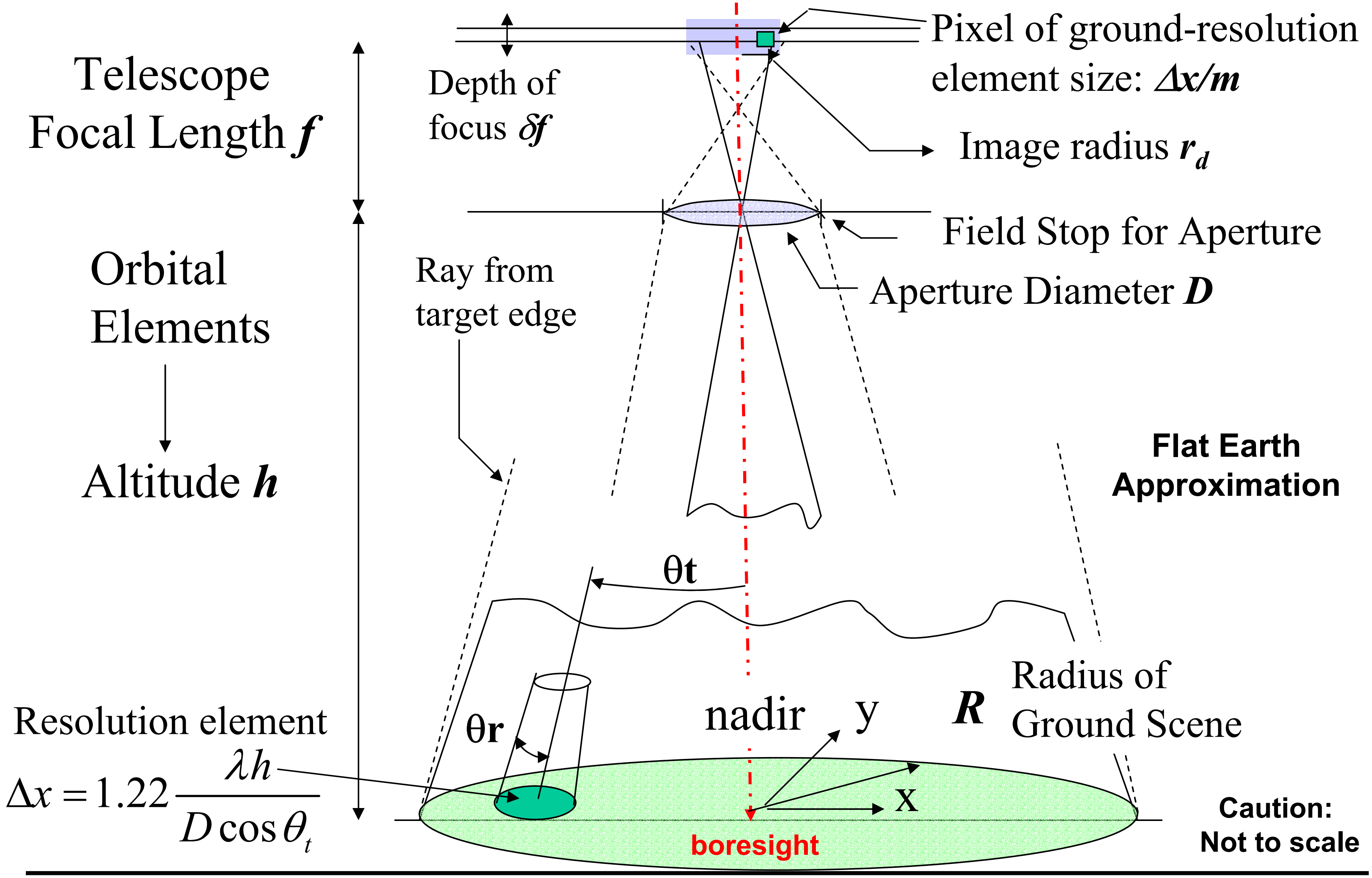
Numerical Aperture $$NA = \frac{1}{2F\#} = \frac{D}{2f}$$

Image brightness is proportional to $1/F^2$

Depth of focus ***δf***: $$\frac{1}{f} = \frac{1}{h} + \frac{1}{f + \delta f}$$

Best optical systems are DIFFRACTION-LIMITED.

Space Based Imaging



Ground Resolution

Ground Resolution Element defined by (Angular Resolution):

$$\theta_r = 1.22\frac{\lambda}{D}$$

(Rayleigh Diffraction Criterion)

Length Normal to Boresight Axis:

$$\Delta x = 1.22\frac{\lambda h}{D\cos\left(\theta_t\right)}$$

Relate this to the 0.3m ground resolution requirement given in the SOW

Assumes a circular aperture

For astronomical imaging, angular resolution is more relevant metric Since our target is faint distant Stars(point source).
1 arcsec=4.8 micro radians

Field-of-View (FOV)

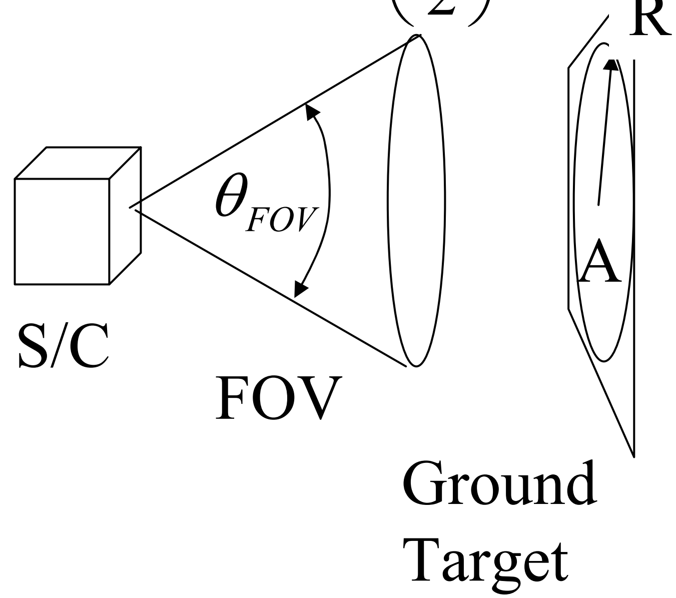
Determines the scope of the image. Defined by angle on the sky/ground we can see in one single image. E.g. "Our FOV is 4x4 arcminutes".

Angular diameter of FOV:

$$\theta_{FOV} = 2 \cdot \tan^{-1}\left(\frac{r_d}{f}\right)$$

Large detector = Large FOV
Long Focal Length ***f*** = Small FOV

$$A = \pi R^2 = \pi (h \tan\left(\frac{\theta}{2}\right))^2$$



Ray Tracing/Optical Train

Ray-Tracing uses first term in paraxial approximation (first order theory). Geometrical Optics is based on two laws of physics:

1. Rectilinear propagation of light in homogeneous media
2. Snell's law of refraction

Optical Elements

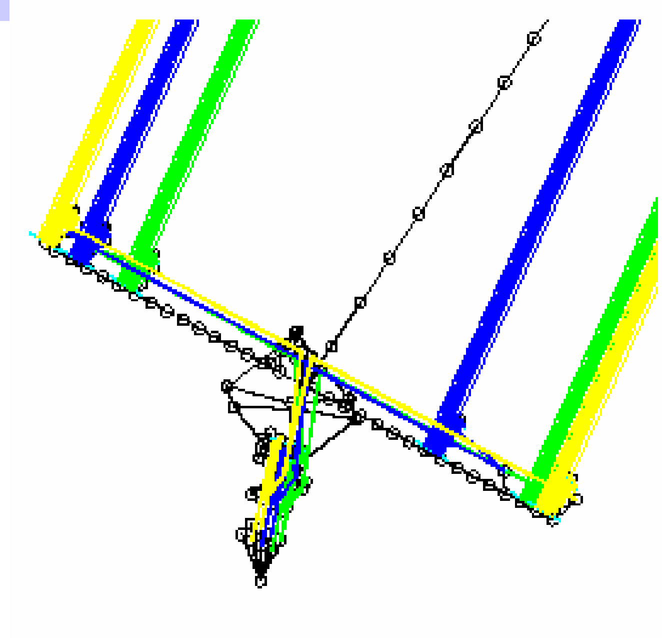
Mirrors
Lenses
Prisms
Filters
Beamsplitters
Compressors
Expanders
Detectors
Delay Lines

SIM Classic
Ray Tracing Diagram

Science

Guide 1

Guide 2



Assumptions: Rays are paraxial, index of refraction n is constant, independent of wavelength (ignore dispersion) and angle.

3 Important Aspects

First Order:
"paraxial approx"
insufficient

Performance of Optical Imaging System

Take into account:
diffraction

Isolation

Angular Resolution determines ability to separate closely spaced objects

Driving Parameters:
D- Aperture Size
λ - wavelength
Optical errors

Ambiguity

Ability to detect a signal in the main lobe from the signals in the side lobes

Driving Parameters:
L_k, ψ - Aperture locations, sparseness of aperture array

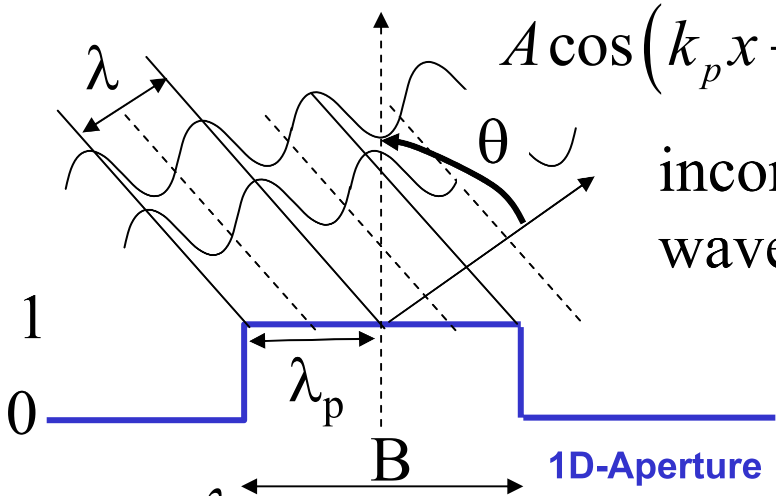
Sensitivity

SNR determines ability to detect faint object

Parameters:
I - Target Irradiance
D - Aperture Size
T - sys temp
QE - detector

Spatial Laplace Transform

$$A\cos\left(k_p x + \omega t + \phi\right)$$



Time Averaging

Aperture samples incoming wavefront and produces an angle dependent intensity

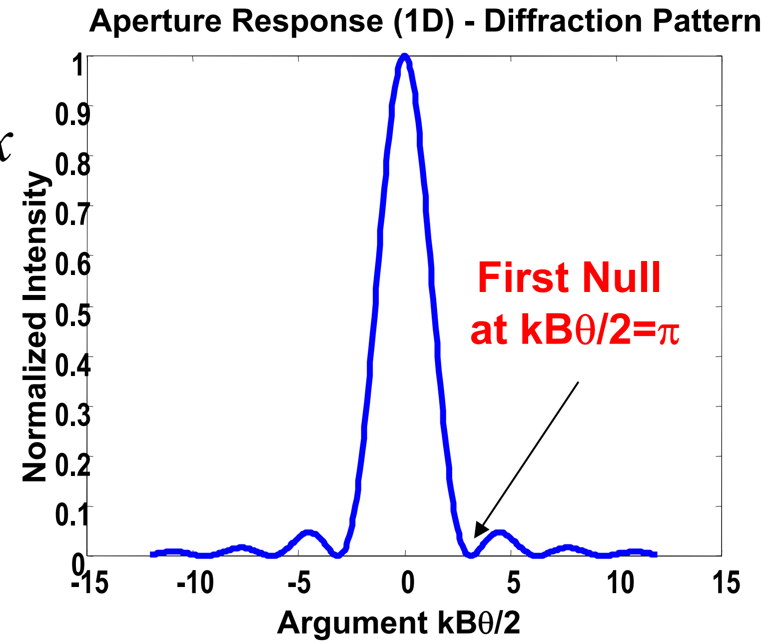
$$\sin\theta = \frac{\lambda}{\lambda_p}$$

$$k = \frac{2\pi}{\lambda}$$

$$k_p = k\sin\theta$$

$$I = \underbrace{\frac{A^2}{2}}_{I_o} \int_{-B/2}^{B/2} \left[\cos(k_p x)\right]^2 dx$$

$$I(\theta) = \left[\frac{\sin(kB\theta/2)}{kB\theta/2}\right]^2 I_o$$



Point-Spread-Function (PSF)

Represents the 2D-spatial impulse response of the optical system.

J_1: First order Bessel Function

Other names:

Fraunhofer diffraction pattern

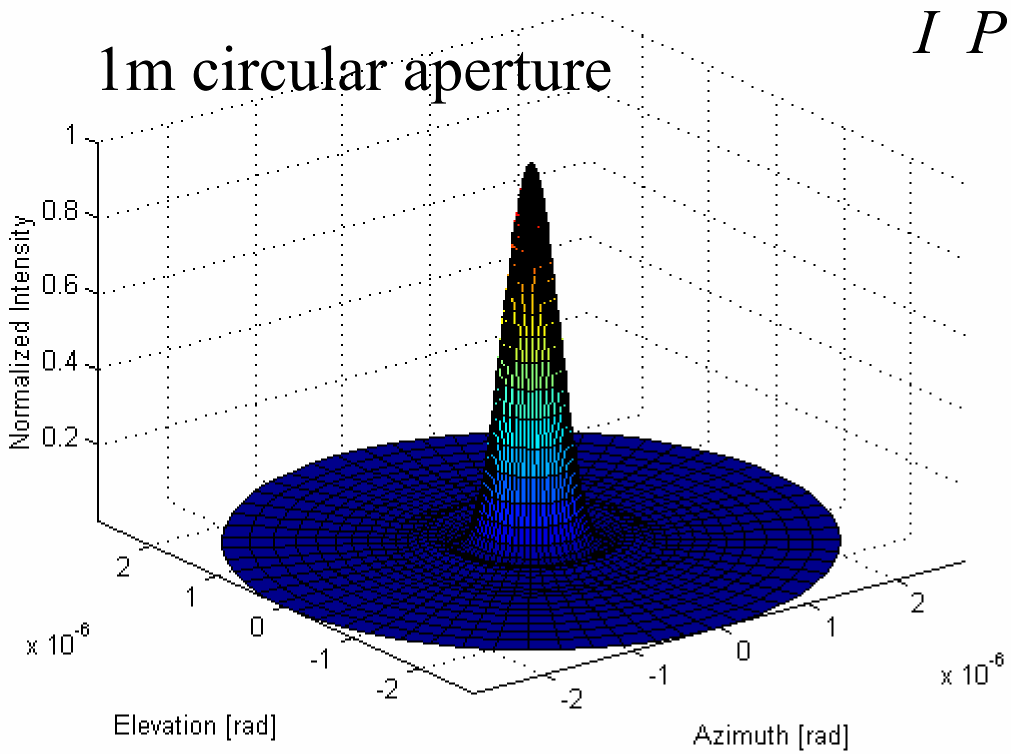
Airy pattern   a=D/2

$$I(P) = |U(P)|^2 = \left[\frac{2J_1(ka\omega)}{ka\omega}\right]^2 I_o$$

where $I_o \propto D^2$

P is a point in the diffraction pattern: P=P(ω,ψ)

1m circular aperture



Normalized PSF for a monolithic, filled, circular aperture with Diameter =1m

Encircled Energy

Diffraction pattern of light passed through a pinhole or from a circular aperture and recorded at the focal plane: Airy Disk

Bessel-Function of n-th order:

$$J_n(x) = \frac{i^{-n}}{2\pi} \int_0^{2\pi} \exp(ix\cos\alpha) \cdot \exp(in\alpha)\, d\alpha$$

Maxima/Minima of $y = \left[\frac{2J_1(x)}{x} \right]^2$

x	y	
0	1	Max
1.22π	0	Min
1.635π	0.0175	Max
2.233π	0	Min

Central ring contains 83.9% of the total energy passing through the aperture.

SNR and Integration Time

Look at the Power = Energy/unit time we receive from ground

Solid Angle FOV: $\omega_d = A / h^2$

Solid angle defining upwelling flux from a resolution element: $\omega_d = A_d \cos\theta_t / (h/\cos\theta_t)^2 \approx \pi\left(\theta_r / 2\right)^2$

Dwell Time: $t_d \propto \tau, \frac{1}{D^2}, I_{gnd}, \ldots$

τ - detector time constant
D - aperture size,
I_{gnd} - ground Irradiance

SNR: = S/N "Optical Link Budget" , see SMAD

IR Imaging systems must be cooled to achieve low noise, use passive cooling or active cooling (cryocoolers).

Static Optical Aberrations

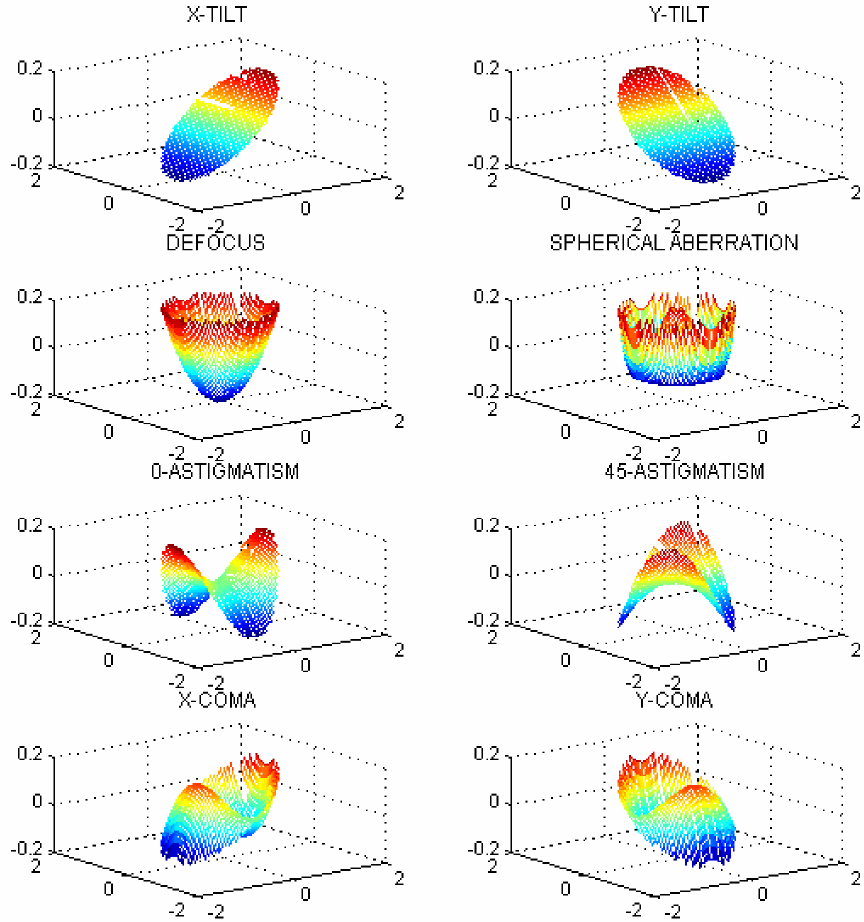
Zernike Polynomials and
Seidel Coefficients

Example: Spherical Aberration

$$I_\nu = \left(\frac{h_\nu}{h_1}\right)^4 Q_\nu^2 \Delta\left(\frac{1}{ns}\right)_\nu$$

See the next page
for definitions

Spherical Aberration, Coma
= change in magnification
throughout the FOV



Also have dynamic errors (WFE RMS)

Static Optical Aberrations (II)

Chromatic Aberration -- usually associated with objective lenses of refractor telescopes. It is the failure of a lens to bring light of different wavelengths (colors) to a common focus. This results mainly in a faint colored halo (usually violet) around bright stars, the planets and the moon. It also reduces lunar and planetary contrast. It usually shows up more as speed and aperture increase. Achromat doublets in refractors help reduce this aberration and more expensive, sophisticated designs like apochromats and those using fluorite lenses can virtually eliminate it.

Spherical Aberration -- causes light rays passing through a lens (or reflected from a mirror) at different distances from the optical center to come to focus at different points on the axis. This causes a star to be seen as a blurred disk rather than a sharp point. Most telescopes are designed to eliminate this aberration.

Coma -- associated mainly with parabolic reflector telescopes which affect the off-axis images and are more pronounced near the edges of the field of view. The images seen produce a V-shaped appearance. The faster the focal ratio, the more coma that will be seen near the edge although the center of the field (approximately a circle, which in mm is the square of the focal ratio) will still be coma-free in well-designed and manufactured instruments.

Astigmatism -- a lens aberration that elongates images which change from a horizontal to a vertical position on opposite sides of best focus. It is generally associated with poorly made optics or collimation errors.

Field Curvature -- caused by the light rays not all coming to a sharp focus in the same plane. The center of the field may be sharp and in focus but the edges are out of focus and vice versa.

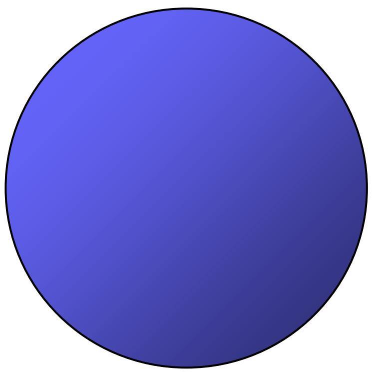
Atmosphere

In Telescope Design account for:

Scattering by aerosols and airborne particles
Scattering proportional to $1/\lambda^4$
Index of refraction of the air is not constant (!)

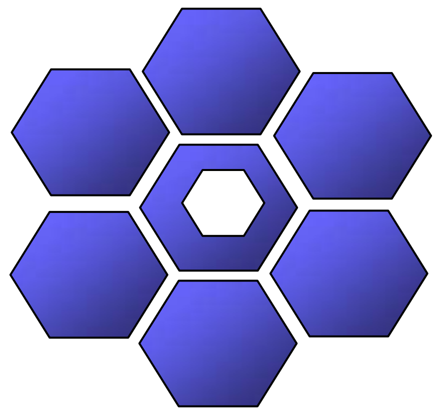
Primary Aperture Types

Monolithic



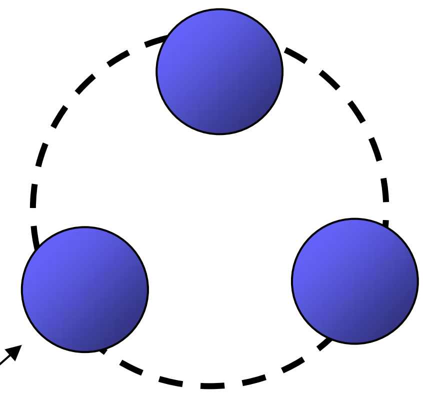
Examples:
Palomar

Segmented



Examples:
NGST,MMT

Sparse



"spangles"

Examples:
SIM, VLT

In your study: consider different aperture types and their effect on the optical image quality, the PSF, resolution, ambiguity and SNR.

Telescope Types (I)

Design Goal: Reduce physical size while maintaining focal length f
Solution: Folded reflective Telescopes

- Refractors
- Newtonian Reflectors
- Cassegrain
  - Two Mirrors
  - Catadioptric System
- Off-axis Systems

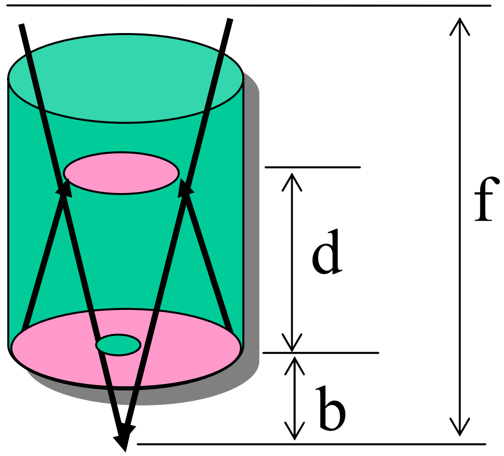
- **Single Mirror(Newtonian) :**

A small diagonal mirror is inserted in the focusing beam. A more accessible focused Spot, but produces a central obscuration in the aperture and off-axis coma

- **Two Mirror Focusing (Cassegrain):**

Improve the system field of view, reduce the package size while maintaining a given Focal length and performance characteristics

Telescope Types(II)-Cassegrain



f = effective focal length, focal length of the system
f1= focal length of primary(positive,concave)
F2= focal length of secondary(negative,convex)
D1,D2= Diameter of primary,secondary mirror

(1)Effective focal length:

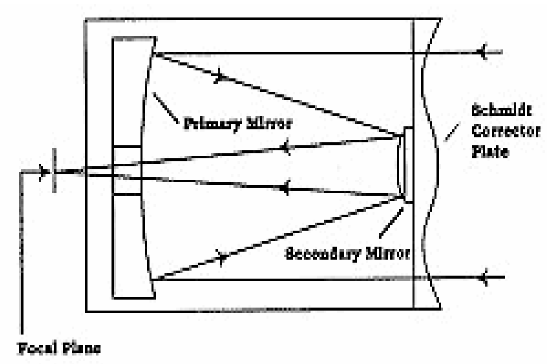
$$f = \frac{f_1 f_2}{f_1 + f_2 - d}$$

(2)Secondary Mirror Apertures

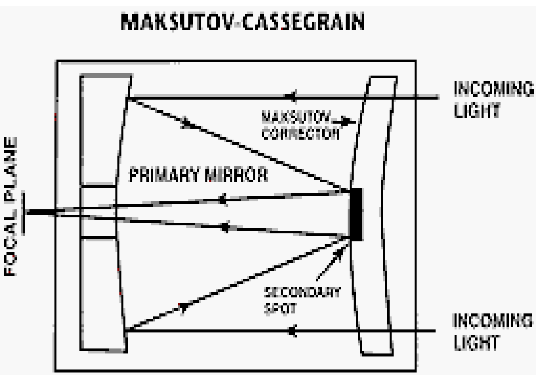
$$D_2 = D_1 (1 - \frac{d}{f_1})$$

System	Primary	Secondary	Comment
Classical Cassegrain	**Paraboloid**	**Hyperboloid**	Off-Axis Performance Suffers
Dall-Kirkham	**Prolate ellipsoid**	**Sphere**	Less Expensive, degraded Off-Axis errors
Ritchey-Chretien	**Hyperboloid**	**Hyperboloid**	Completely corrected spherical aberration & coma (expensive)
Pressman-Camichel	**Sphere**	**Oblate epllipsoid**	

Telescope Types(III)-Catadioptric



Schmidt-Cassegrain the light enters through a thin aspheric Schmidt correcting lens, then strikes the spherical primary mirror and is reflected back up the tube and intercepted by a small secondary mirror which reflects the light out an opening in the rear of the instrument. **Compact(F# f/10-f/15), Correcting Lens eliminates Spherical aberration,coma, Astigmatism, Image Field Curvature at the expense of central obstruction,chromatic error(from refractive lense)**



MAKSUTOV-uses a thick meniscus correcting lens with a strong curvature and a secondary mirror that is usually an aluminized spot on the corrector. The secondary mirror is typically smaller than the Schmidt's giving it slightly better resolution for planetary observing. Heavier than the Schmidt and because of the thick correcting lens takes a long time to reach thermal stability at night in larger apertures (over 90mm). Typically is easier to make but requires more material for the corrector lens than the Schmidt-Cassegrain.

Detectors

Fundamentally three types:

(1) Photographic Plate/Film
(2) Electronic Detector (e.g. CCD)
(3) Human Eye

CCD most important for remote sensing (electronic transmission)

Detector field area: $$A_d = \pi r_d^2$$

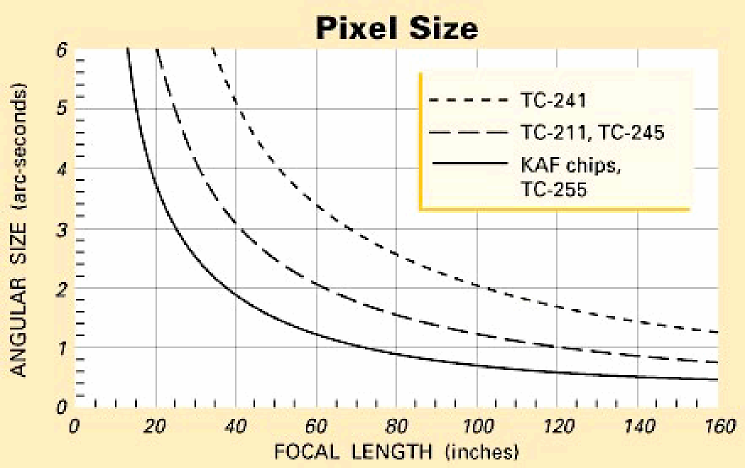
Depth of Focus: $$\delta f = \pm 2\lambda (F\#)^2$$

<u>Sample CCD Design Parameters:</u>

Format: 2048(V) x 1024 (H)
Pixel Shape: Square
Pixel Pitch: 12 μm
Channel Stop Width: 2.5 μm

Quantum Efficiency: >0.60
Full Well condition: >100,000 e-
Dark Current: < 1nAmp/cm^2

Optimizing a CCD Imaging System (I)



- Pixel Size: $d = (2\theta_r f)Q = (2.44\lambda f / D)Q$
  Q(quality factor)=1/2 used to avoid undersampling

- # of pixels <= FOV

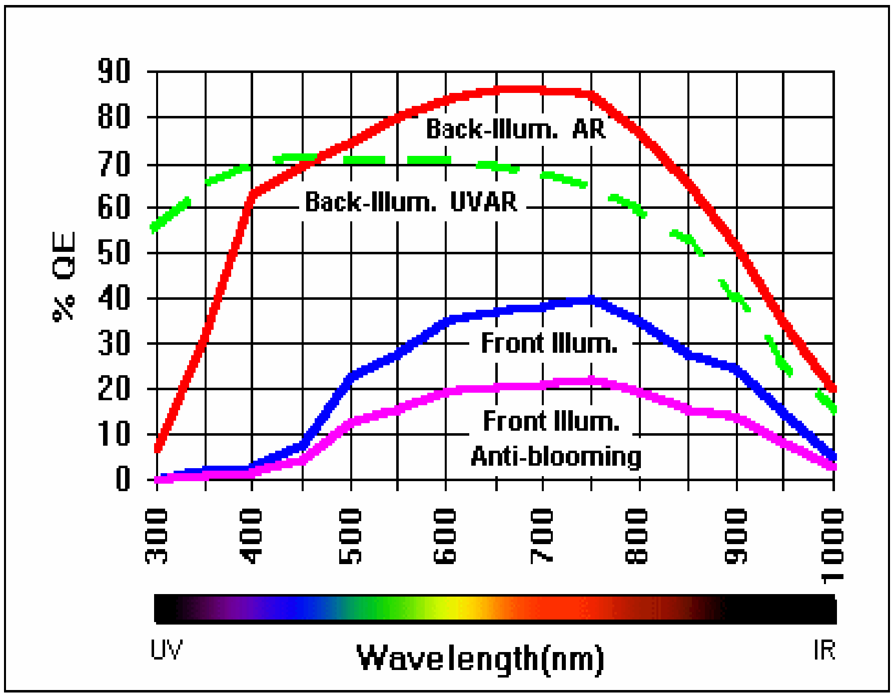
- Sensitivity :

Rather than the total amount of signal in an image (which depends on gain in the camera's electronics), sensitivity is the signal-to-noise ratio (S/N) obtained with a given exposure time. The S/N is a measure of quality; the higher the ratio, the less gritty an image will appear

- A very good deep sky object at least 25 S/N.
- Smaller pixel(9µm)=>longer exposure time (lower sensitivity)
  a faint deepsky object may be **oversampled**
- Larger pixel(24µm)=> greater sensitivity, **undersampled** for bright source.

Optimizing a CCD Imaging System (II)

•Anti-blooming: helps protect against the objectionable streaks that occur when bright objects saturate the CCD, causing an excess charge to bleed down a column of pixels. This feature can, however, produce side effects like increased dark current and reduced sensitivity.



• Quantum Efficiency (QE):

Q.E. of a sensor describes its response to different wavelengths of light (see chart). Standard front-illuminated sensors, for example, are more sensitive to green, red, and infrared wavelengths (in the 500 to 800 nm range) than they are to blue wavelengths (400 - 500 nm). **Note** Back-illuminated CCDs have exceptional quantum efficiency compared to front-illuminated CCDs.