16.50 Lecture 23

Subject: Exhaust nozzles

Like the inlet, the nozzle can range from very simple to quite complex. A simple fixed-area convergent form usually suffices for subsonic aircraft except when jet noise suppression is required, while a complex variable-area convergent-divergent device is essential for adequate performance in supersonic aircraft. In either case, the functions of the nozzle are two:

a) Provide the required throat area to match the mass flow and exit conditions of the engine.

b) Efficiently expand the high-pressure, high-temperature gases at the engine exhaust to atmospheric pressure, converting the available thermal energy to kinetic energy.

Throat Area:

The required throat area is determined by conservation of mass. We had from Lecture 20

$$\dot{m} = \Gamma \overline{m}_2 \frac{p_{t2} A_2}{\sqrt{R T_{t2}}}$$

and if the throat (station 7) is choked,

$$\dot{m} = \Gamma \frac{p_{t7} A_7}{\sqrt{R T_{t7}}} = \Gamma \frac{p_{t5} A_7}{\sqrt{R T_{t5}}}$$

Equating and using $\frac{p_{t5}}{p_{t2}} = (\tau_c \tau_t)^{\frac{\gamma}{\gamma-1}}$ and $\frac{T_{t5}}{T_{t2}} = \vartheta \tau_t$, we can calculate

$$\frac{A_7}{A_2} = \frac{\overline{m}_2(M_2)}{\tau_c^{\frac{\gamma}{\gamma-1}} \tau_t^{\frac{\gamma+1}{2(\gamma-1)}}} \sqrt{\vartheta}$$

The ratios in the expression for A_7/A_2 are determined by cycle analysis, as outlined previously, so from this expression we can find the nozzle area ratio as a function of the engine parameters. As we have seen in our discussion of the matching of components, once the nozzle area is set, the operating point of the engine depends only on the turbine temperature ratio θ.

Exit Area:

The ratio of exit area to throat area required for ideal expansion can be found from the usual compressible channel flow relations. Thus the exit Mach number is set by the stagnation pressure of the jet and the ambient pressure

$$\frac{p_{te}}{p_o} = \frac{p_{te}}{p_e} = \left(1 + \tfrac{\gamma-1}{2} M_e^{\,2}\right)^{\frac{\gamma}{\gamma-1}}$$

where
$$\frac{p_{te}}{p_0} = \vartheta_0 \tau_c \tau_t$$

and the area ratio is then set by this Mach number,

$$\frac{A_e}{A_7} = \frac{1}{M_e}\left[\frac{1 + \frac{\gamma-1}{2} M_e^{\,2}}{\frac{\gamma+1}{2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Since p_{te}/p_0 involves θ_0, M_e for this matched condition does depend on flight Mach number, and so does then A_e/A_7. This is important for supersonic engines, as we discuss below.

These relations taken together serve to define the geometry of an ideally expanded nozzle. If the nozzle is not ideally expanded, the behavior is quite like that of the rocket nozzle at off-design conditions, as discussed earlier.

Effects of nozzle mismatching: Subsonic vs. supersonic

We can use Eqs. (15) and (18) of Lecture 19(b) to calculate the thrust of engines whose nozzles are respectively pressure-matched or truncated at the sonic point. We illustrate this for two different engine designs, one subsonic and one supersonic:

Case 1: $M_0=0.85$ $(\theta_0=1.1445)$
Case 2: $M_0=2$ $(\theta_0=1.8)$

For both cases, we take $\theta_t=6.25$ and $M_2=0.6$ (or $\overline{m}_2 = 0.8416$). We also assume the compressor ratio is that which gives maximum thrust in each case ($\tau_c = \sqrt{\vartheta_t/\vartheta_0}$). The results are shown below:

M_0	τ_c	τ_t	A_{Throat}/A_2	A_e/A_2	$(\varphi_2)_{Matched}$	$(\varphi_2)_{Truncated}$
0.85	2.1844	0.7831	0.2658	0.2700	0.9682	1.4991
2	1.3889	0.8880	0.7093	0.7190	0.9682	0.7666

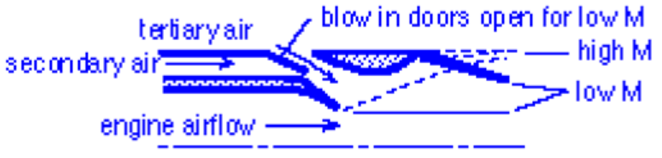
There is little effect (8%) on thrust for the subsonic nozzle when the nozzle is truncated at its throat, but the effect is a 26% thrust reduction in the supersonic case. This is why such engines carry an adjustable convergent-divergent nozzle.

Additional Requirements:

The exhaust nozzle may have to meet a number of other requirements. Some are:

a) Variable area for afterburning, to increase throat area in proportion to the square root of the temperature after afterburning:

a) variable geometry ejector nozzle



b) ejector nozzle with blow in doors for tertiary air

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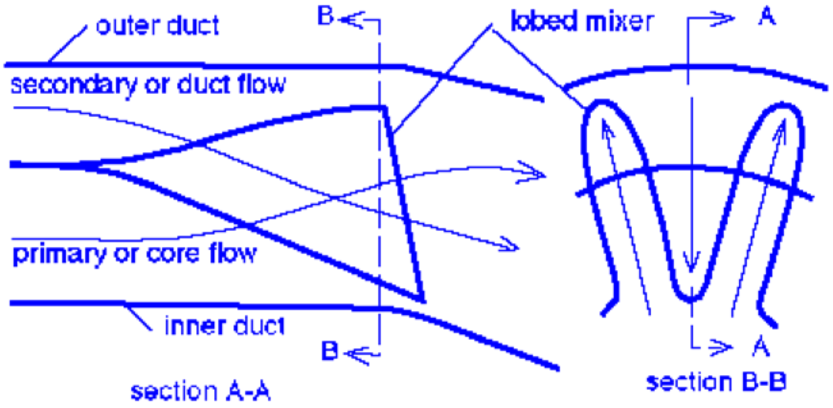
Such nozzles have been built in a number of forms. At the top is the relatively simple type used on the F-111 and F-15 engines, which have top flight Mach numbers of the order of 2. At the bottom is the type used on the SR-71, which reaches 3.5 or thereabouts.

b) Noise suppression

As we shall see in Lecture 37, the principal way to decrease jet noise is to lower the jet velocity for a given thrust. The turbofan is the most efficient way to do this, but for some applications it is not practical to use a fan. In this case there is the desire to increase the mass flow rate of the jet, by mixing in additional air that lowers the velocity for a given total momentum. One way to do this is the lobed mixer:



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Lobed mixers are also used sometimes in turbofans to help equalize the core and bypass stream velocities, and hence increase performance (by 3-4%). The improvement is, however, less than if the velocity changes were isentropic.

c) Thrust-vectoring, as for advanced VTOL aircraft.

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