16.50 Propulsion Systems
Spring 2011

Quiz 2 May 18, 2011

Two hours, open book, open notes

TRUE-FALSE QUESTIONS

Justify your answer in no more than two lines.
4 points for correct answer and explanation
2-3 points for a correct answer with only partially correct explanation
1-2 points for an incorrect answer with some valid argument
0 for an incorrect answer with an incorrect explanation, or any answer with no explanation

Statement	True	False
1. For a turbojet, a high $\theta_t = {T_{t4}}/{T_{t0}}$ gives a high thermodynamic efficiency η_{th} at any compression ratio π_c. *One can have a ${T_{t4}}/{T_{t0}}$ with very small pressure ratio in the cycle. This gives a Brayton cycle low efficiency.*		√
2. The pressure ratio of the turbine does not change when the pilot changes the fuel/air ratio f. $\tau_t = \left(\frac{A_7}{A_4}\right)^{\frac{2(\gamma-1)}{\gamma+1}}$ *is fixed by geometry (due to double choking of the flow).*	√	
3. If the throat area A_7 of a turbojet decreases due to some obstruction, the compressor operating line moves closer to the stall line. *The compressor operating line is* $\bar{m}_2 = \frac{A_7}{A_4}\pi_c\sqrt{\frac{\frac{1-\tau_t}{\gamma-1}}{\pi_c^{\frac{\gamma}{\gamma}}-1}}$, *and* $\tau_t = \left(\frac{A_7}{A_4}\right)^{\frac{2(\gamma 0 1)}{\gamma+1}}$. *If* $A_7\uparrow, \tau_t\downarrow, \bar{m}_2\downarrow$.	√	
4. The bypass ratio α of a turbofan engine is fixed by the geometry, and does not change with operating conditions. *It does not vary with* θ.		√
5. For a fixed compressor face Mach number M_2, the cowl lip of a subsonic inlet would choke if its area A_1 were less than $\bar{m}_2(M_2)A_2$, where $\bar{m}_2$ is the non-dimensional flow factor. *Equate flows when A_1 chokes:* $\bar{m}_2\frac{P_{t2}A_2}{\sqrt{T_{t2}}} = \frac{P_{t0}A_1}{\sqrt{T_{t0}}}$, *and since* $P_{t2} = P_{t0}$, $T_{t2} = T_{t0}$, $\bar{m}_2 A_2 = A_1$	√	
6. The Euler equation is only valid for ideal, isentropic flow. *It is a mechanical work balance, so it is not sensitive to non-idealities.*		√
7. The stall line on a compressor map can be pre-determined by flow matching conditions even before the specific compressor has been selected. *The working line is pre-determined, but the stall line depends on compressor details.*		√
8. In a multi-stage turbine in which each stage has the same isentropic efficiency η_{stage}, the overall turbine isentropic efficiency η_T is greater than η_{stage}. *The inefficiency $1-\eta_{stage}$ of each stage means that extra heat is deposited in the flow by each stage, and the subsequent stages convert some of it to work by expansion.*	√	

9. The nitrogen oxides produced in the primary zone of a jet engine burner are largely destroyed by the cooler secondary air that is injected downstream. *The destruction reactions are very slow.*		√
10. A quadrupole made up of four monopoles emits less acoustic power than would each of the monopoles separately. *There are partial cancellations between positive and negative monopoles.*	√	

PROBLEM 1 (30 points)

The design of a certain turbofan engine is such that the turbine inlet temperature at takeoff on a standard day ($T_0 = 288\ K, P_0 = 1\ atm$ is $1650\ K$, and the compressor-face Mach number is $M_2 = 0.5$. The compressor is designed to provide maximum thrust at that condition. A set of such engines provides the required thrust (including margin) for takeoff of a passenger jet plane.

Consider now a "hot day" situation ($T_0 = 305\ K, P_0 = 1\ atm$) for the same plane, with the same load and at the same take-off Mach number. How will the following quantities change from their design values?:

- Thrust F
- Normalized thrust $\varphi = \frac{F}{P_{t2}A_2}$
- Normalized peak temperature $\theta = {}^{T_{t4}}/_{T_{t0}}$
- Peak temperature T_{t4}
- Normalized flow rate $\overline{m}_2$
- Flow rate $\dot{m}$
- Fuel flow rate $\dot{m}_f$
- Compressor pressure ratio π_c

Same plane weight W, same M_0, gives same lift L, and assuming same aerodynamic L/D, same drag D. Therefore, same thrust.

$$F' = F$$

For the same M_0 and P_0, same P_{t0}, so same normalized thrust.

$$\varphi' = \frac{F'}{P'_{t0}A_2} = \varphi = \frac{F}{P_{t0}A_0}$$

Now, from lesson 18b, $\varphi = \varphi(\theta, M_0)$, hence same θ:

$$\theta' = \frac{T'_{t4}}{T'_{t0}} = \theta = \frac{T_{t4}}{T_{t0}}$$

Since $M_0' = M_0, \frac{T_{t0}'}{T_{t0}} = \frac{T_0'}{T_0} = \frac{305}{288} = 1.059$. *Therefore,* $\frac{T_{t4}'}{T_{t4}} = \frac{T_{t0}'}{T_{t0}} = 1.059$.

$$T_{t4}' = 1.059 \times 1650 = 1747\ K$$

The normalized flow rate $\overline{m}_2$ *, the compressor ratios* π_c *,* τ_c*, and the ratio* $\frac{fh}{c_p T_{t0}}$ *depend exclusively on* θ*, so*

$$\pi_c' = \pi_c$$
$$\overline{m}_2' = \overline{m}_2$$

Now, $\dot{m} = \overline{m}_2 \Gamma \frac{P_{t0} A_2}{\sqrt{R T_{t0}}}$*, and so*

$$\frac{\dot{m}'}{\dot{m}} = \sqrt{\frac{T_{t0}}{T_{t0}'}} = \frac{1}{\sqrt{1.059}} = 0.972$$

Also, $f = \frac{\dot{m}_f}{\dot{m}}$*, so* $\frac{\dot{m}_f'}{\dot{m}_f} = \frac{f'}{f}\frac{\dot{m}'}{\dot{m}} = \frac{T_{t0}'}{T_{t0}}\sqrt{\frac{T_{t0}}{T_{t0}'}} = \sqrt{\frac{T_{t0}'}{T_{t0}}}$

$$\frac{\dot{m}_f'}{\dot{m}_f} = \sqrt{1.059} = 1.029$$

PROBLEM 2 (30 points)

In designing one of the identical stages of a compressor, we wish to maximize the stage temperature rise ΔT_t, so as to minimize the number of stages, while limiting the stage loading to avoid excessive losses. Assume a 50% reaction design, with the axial velocity *w* determined by a compressor-face Mach number $M_2 = 0.5$, and an inlet total temperature $T_{t0} = 250\ K$.

a) Show from the Euler equation that high ΔT_t per stage is favored by high wheel spin $\omega \bar{r}$ and low stator exit angle β_1 (or, for this design, $\beta_2' = \beta_1$). Assume the wheel speed is as high as allowed by hoop stress limitations on the rim (assumed to be self-sustaining, namely, the blade centrifugal pull is compensated by the disk tension). The rim material is a Titanium alloy with working stress $\sigma = 6 \times 10^8\ Pa$, and density $\rho = 4500\ kg/m^3$. Take the blade ratio ${}^{r_H}/_{r_T} = 0.8$, so that ${}^{\bar{r}}/_{r_H} = 9/8$. Calculate $\omega \bar{r}$.

Axial velocity: $w = M_2\sqrt{\gamma R T_2} = M_2 \sqrt{\frac{\gamma R T_{t0}}{1 + \frac{\gamma - 1}{2} M_2^2}}$

$$w = 0.5\sqrt{\frac{1.4 \times 287 \times 250}{1 + 0.2 \times 0.5^2}} = 309.3\, m/s$$

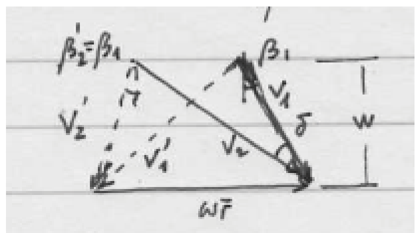
Wheel speed: or self-sustaining rim, $\sigma = \rho(\omega^2 r_H^2)$, and $\bar{r} = 9/8(r_H)$, so

$$\omega\bar{r} = \frac{9}{8}\omega r_H = \frac{9}{8}\sqrt{\frac{\sigma}{\rho}} = \frac{9}{8}\sqrt{\frac{6 \times 10^8}{4500}}$$

$$\boxed{\omega\bar{r} = 410.8\, m/s}$$

Euler equation: $c_p \Delta T_t = \omega\bar{r}(\omega\bar{r} - 2w\tan\beta_1)$

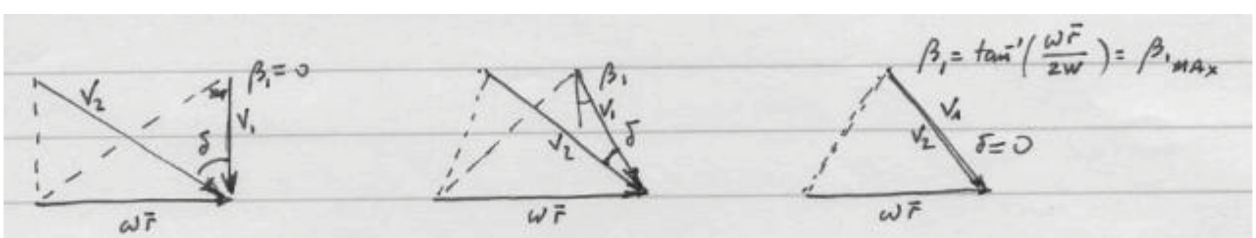
Because $v_1 = w\tan\beta_1$, $v_2 = \omega\bar{r} - w\tan\beta_1$, so $v_2 - v_1 = \omega\bar{r} - 2w\tan\beta_1$



Euler's equation shows ΔT_t increases with $\omega\bar{r}$ and decreases with β_1, as stated.

b) Draw the velocity triangle and show that the flow turning angle δ (the angle between V_1 and V_2, or between V_1' and V_2') increases as β_1 decreases. Values of β_1 that are too small will therefore lead to excessive blade losses, and possibly to stall. Choose the smallest β_1 that keeps $\delta \geq 15°$.

Let us consider three values of β_1: zero, a medium value, and the maximum that still makes $\omega\bar{r} - 2w\tan\beta_1 > 0$:



It is clear that as β_1 increases (left to right), δ decreases. So, the smallest β_1 angles give the highest blade loading (most flow turning).

If δ is prescribed, what is β_1? We have, from geometry,

$$\delta = \tan^{-1}\left(\frac{\omega\bar{r} - w\tan\beta_1}{w}\right) - \beta_1$$

Define t = $\tan\beta_1$ *,* $\frac{w}{\omega\bar{r}} = \phi$*:*

$$\delta = \tan^{-1}\left(\frac{1}{\phi} - t\right) - \tan^{-1} t$$

Take the tangent of both sides:

$$\tan\delta = \frac{\frac{1}{\phi} - 2t}{1 + \left(\frac{1}{\phi} - t\right)t}$$

This can be rearranged as a quadratic equation for t:

$$t^2 - \left(\frac{1}{\phi} + \frac{2}{\tan\delta}\right) + \frac{1}{\phi\tan\delta} - 1 = 0$$

With solution:

$$t = \frac{1}{2\phi} + \frac{1}{\tan\delta} \pm \sqrt{\frac{1}{4\phi^2} + \frac{1}{\tan\delta} + 1}$$

In our case, $\delta = 15°$ *(using the negative square root, the positive makes* $\omega\bar{r} - 2w\tan\beta_1 < 0$*).*

$$t = \tan\beta_1 = \frac{410.8}{2 \times 309.3} + \frac{1}{\tan 15°} - \sqrt{\left(\frac{410.8}{2 \times 309.3}\right)^2 + \frac{1}{\tan^2 15°} + 1}$$

$$\boxed{\beta_1 = 25.44°}$$

<u>Note:</u> This can be also found by direct trial-and-error on

$$\delta = \tan^{-1}\left(\frac{410.8}{309.3} - \tan\beta_1\right) - \beta_1$$

c) With these choices, calculate the temperature rise ΔT_t per stage. How many stages would be required to achieve an overall pressure ratio $\pi_c = 21$?

For $\omega\bar{r} = 410.8\ m/s$, $w = 309.3\frac{m}{s}$, $\beta_1 = 25.44°$*. Euler gives:*

$$\Delta T_t = \frac{410.8(410.8 - 2 \times 309.3 \times 0.04757)}{1005}$$

$$(\Delta T_t)_{stage} = 47.6\ K$$

If $\pi_c = 21$, $\tau_c = \pi_c^{\frac{\gamma-1}{\gamma}} = 21^{\frac{1}{3.5}} = 2.3865$

The number of stages is:

$$N = \frac{T_{t0}(\tau_c - 1)}{(\Delta T_t)_{stage}} = \frac{250(2.3865 - 1)}{47.6} = 7.28$$

Rounding up:

$$\boxed{N = 8}$$

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