16.410-13 Recitation 10 Problems

Problem 1: Simplex Method

Part A

Solve the following two linear programs using the simplex method.

LP 1

$$\begin{aligned} \text{maximize} \quad & 3x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 4 \\ & 2x_1 + x_2 \leq 5 \\ & x_1, x_2 \geq 0. \end{aligned}$$

LP 2

$$\begin{aligned} \text{minimize} \quad & -2x_1 + x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \leq 6 \\ & 3x_1 + 2x_2 \leq 12 \\ & x_1, x_2 \geq 0. \end{aligned}$$

Part B

Consider the case when all the coefficients corresponding to the variables (and the slack variables) of the topmost row of your tableau equals to zero. What would that imply? Can you find an example?

Problem 2: Transportation

Consider a network of mines/factories each of which use others products to produce their own. For instance, the iron ore produced by an iron ore mine is used to produce steel in a steel mill. This steel is used to produce mine wagons, which are then used by the iron ore mines. You would like to optimally coordinate the logistics operations between these mines/factories.

Assume that there are n factories, which are connected with roads. Nodes are assumed to be one way, since you are using certain transportation companies that only operate one way between the factories. A road from factory i to factory j is described by the pair (i, j). Denote the set of all roads by $\mathcal{A}$. Each (directed) road is operated by a single transportation company. The company operating road (i, j) charges $c_{i,j}$ dollars per each pound of cargo. Moreover, the company only has resources (e.g., trucks, trains) to carry up to $u_{i,j}$ tons of cargo per hour. Each factory k generates a certain product at a rate $b^{k,l}$ tons per hour that needs to reach factory l, either through a direct road (k, l) or through some other path in the road network. Products from the same origin can be split into parts and transported through different paths.

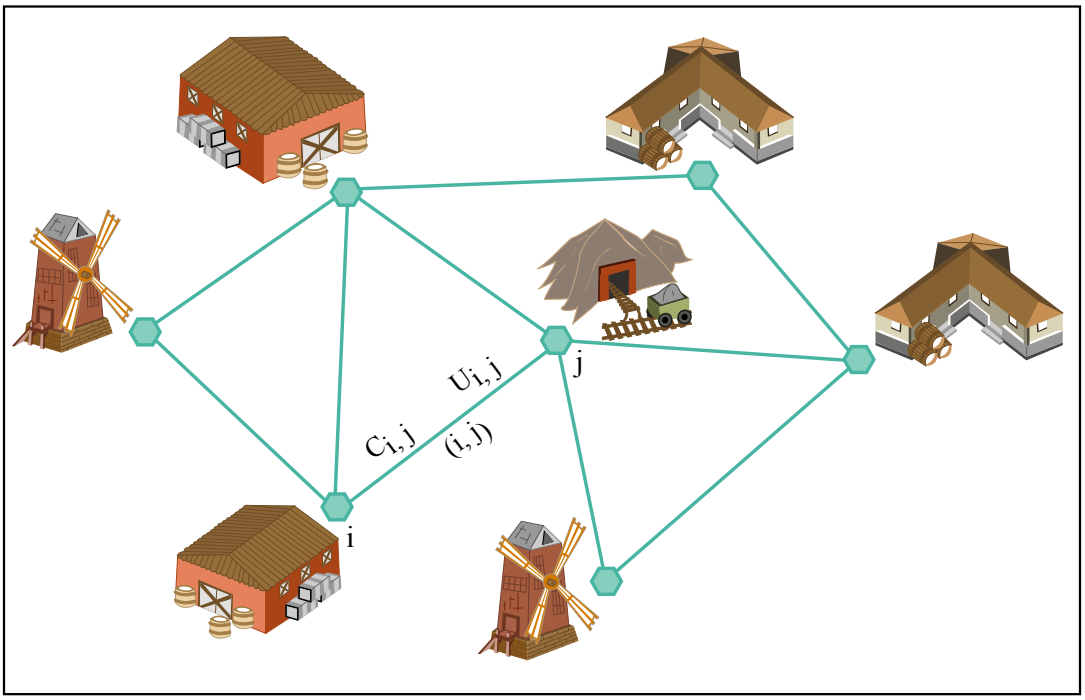

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Figure 1: Multicommodity flow

Formulate this problem as a linear programming problem. This problem is called the *multicommodity flow problem*, extensively studied in operations research. Similar problems are studied, e.g., for the optimization of future space logistics.

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