16.36 Communication Systems Engineering
Spring 2009

Routing in Data Networks

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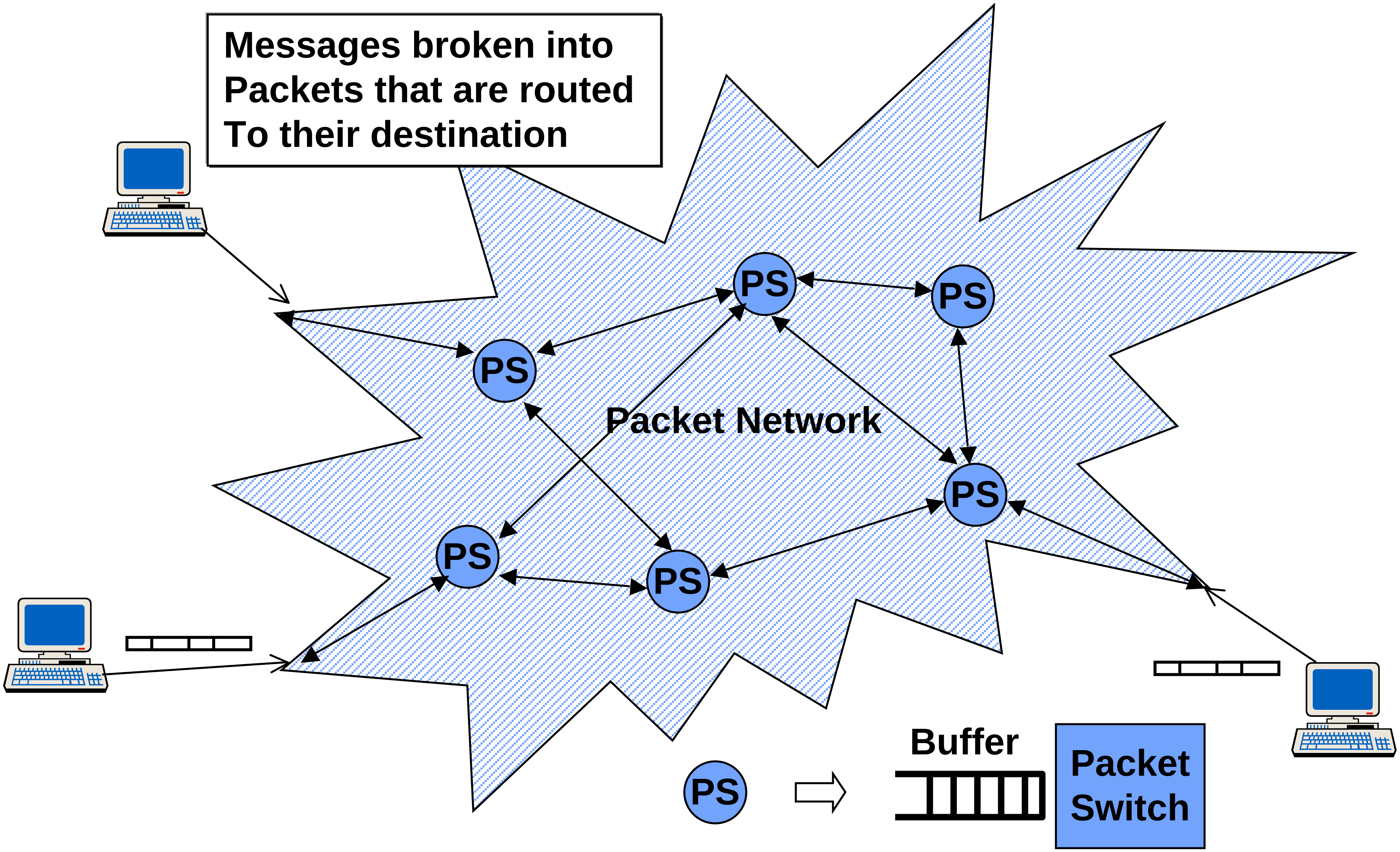
Packet Switched Networks

Messages broken into Packets that are routed To their destination



Routing

- **Must choose routes for various origin destination pairs (O/D pairs) or for various sessions**
  - **Datagram routing: route chosen on a packet by packet basis**

    Using datagram routing is an easy way to split paths

  - **Virtual circuit routing: route chosen a session by session basis**
  - **Static routing: route chosen in a prearranged way based on O/D pairs**

Broadcast Routing

- Route a packet from a source to all nodes in the network

- Possible solutions:

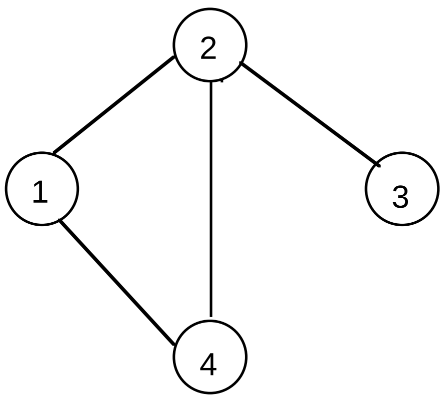
  – Flooding: Each node sends packet on all outgoing links

    Discard packets received a second time

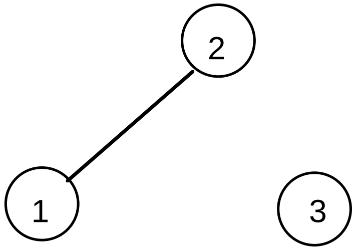
  – Spanning Tree Routing: Send packet along a tree that includes all of the nodes in the network

Graphs

- **A graph G = (N,A) is a finite nonempty set of nodes and a set of node pairs A called arcs (or links or edges)**



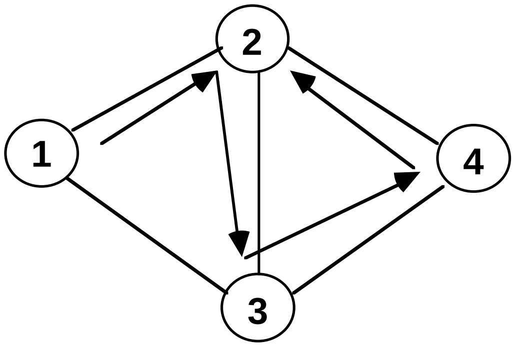
N = {1,2,3,4}
A = {(1,2),(2,3),(1,4),(2,4)}



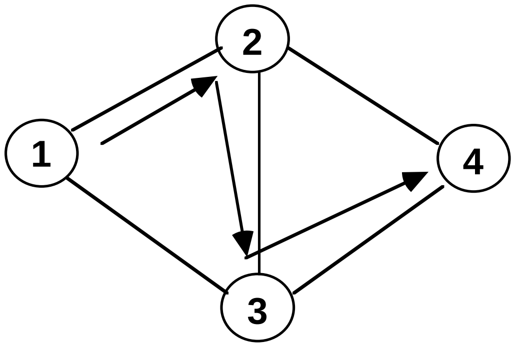
N = {1,2,3}
A = {(1,2)}

Walks and paths

- A walk is a sequence of nodes $(n_1, n_2, ..., n_k)$ in which each adjacent node pair is an arc
- A path is a walk with no repeated nodes



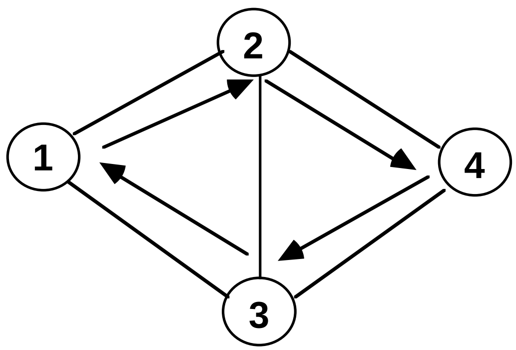
Walk (1,2,3,4,2)



Path (1,2,3,4)

Cycles

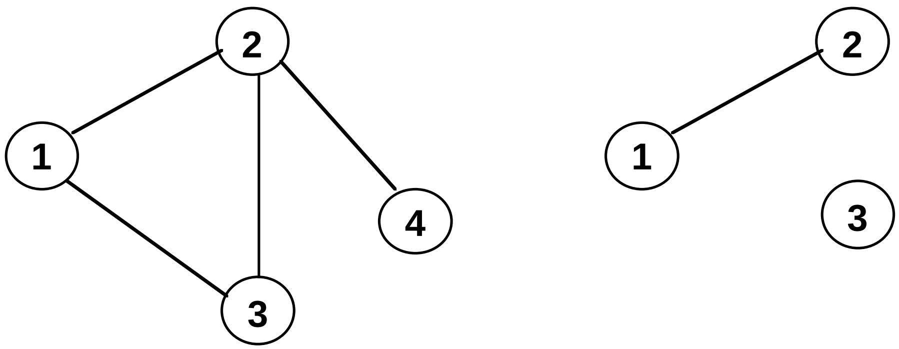
- A cycle is a walk (n_1, n_2,...,n_k) with n_1 = n_k, k > 3, and with no repeated nodes except n_1 = n_k



Cycle (1,2,4,3,1)

Connected graph

- A graph is connected if a path exists between each pair of nodes

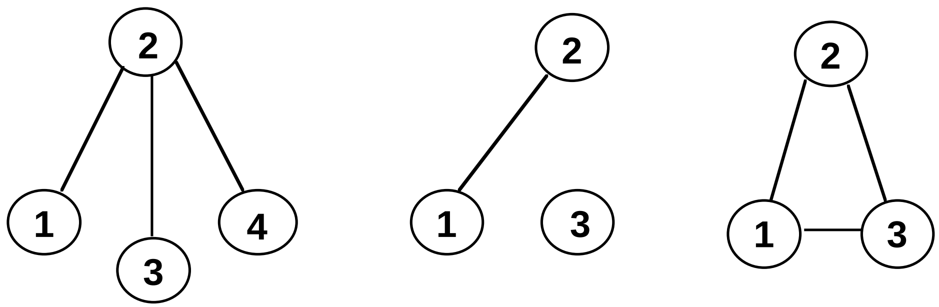


Connected Unconnected

- An unconnected graph can be separated into two or more connected components

Acyclic graphs and trees

- An acyclic graph is a graph with no cycles
- A tree is an acyclic connected graph



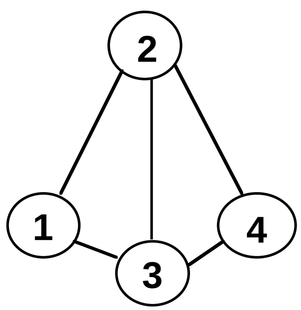
Acyclic, connected | unconnected not tree | Cyclic, not tree

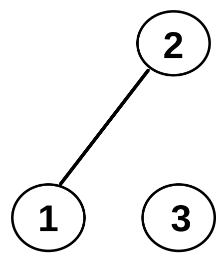
- The number of arcs in a tree is always one less than the number of nodes
  - Proof: start with arbitrary node and each time you add an arc you add a node $\Rightarrow$ N nodes and N-1 links. If you add an arc without adding a node, the arc must go to a node already in the tree and hence form a cycle

Sub-graphs

- G' = (N',A') is a sub-graph of G = (N,A) if
  - 1) G' is a graph
  - 2) N' is a subset of N
  - 3) A' is a subset of A

- One obtains a sub-graph by deleting nodes and arcs from a graph
  - Note: arcs adjacent to a deleted node must also be deleted



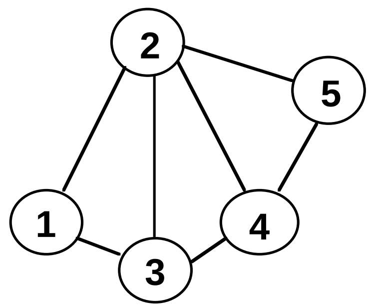


- Graph G

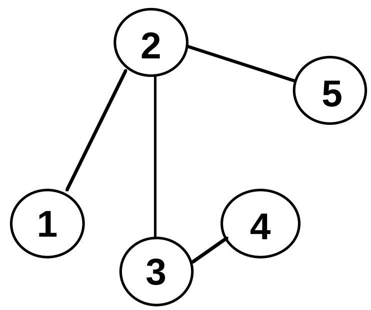
Subgraph G' of G

Spanning trees

- T = (N',A') is a spanning tree of G = (N,A) if
  - T is a sub-graph of G with N' = N and T is a tree



Graph G



Spanning tree of G

Spanning trees

- Spanning trees are useful for disseminating and collecting control information in networks; they are sometimes useful for routing
- To disseminate data from Node n:
  - Node n broadcasts data on all adjacent tree arcs
  - Other nodes relay data on other adjacent tree arcs
- To collect data at node n:
  - All leaves of tree (other than n) send data
  - Other nodes (other than n) wait to receive data on all but one adjacent arc, and then send received plus local data on remaining arc

General construction of a spanning tree

- **Algorithm to construct a spanning tree for a connected graph G = (N,A):**

  1) Select any node n in N; N' = {n}; A' = { }

  2) If N' = N, then stop

  T=(N',A') is a spanning tree

  3) Choose (i,j) $\in$ A, i $\in$ N', j $\notin$ N'

  N' := N'$\cup${j}; A' := A'$\cup${(i,j)}; go to step 2

- **Connectedness of G assures that an arc can be chosen in step 3 as long as N' $\neq$ N**

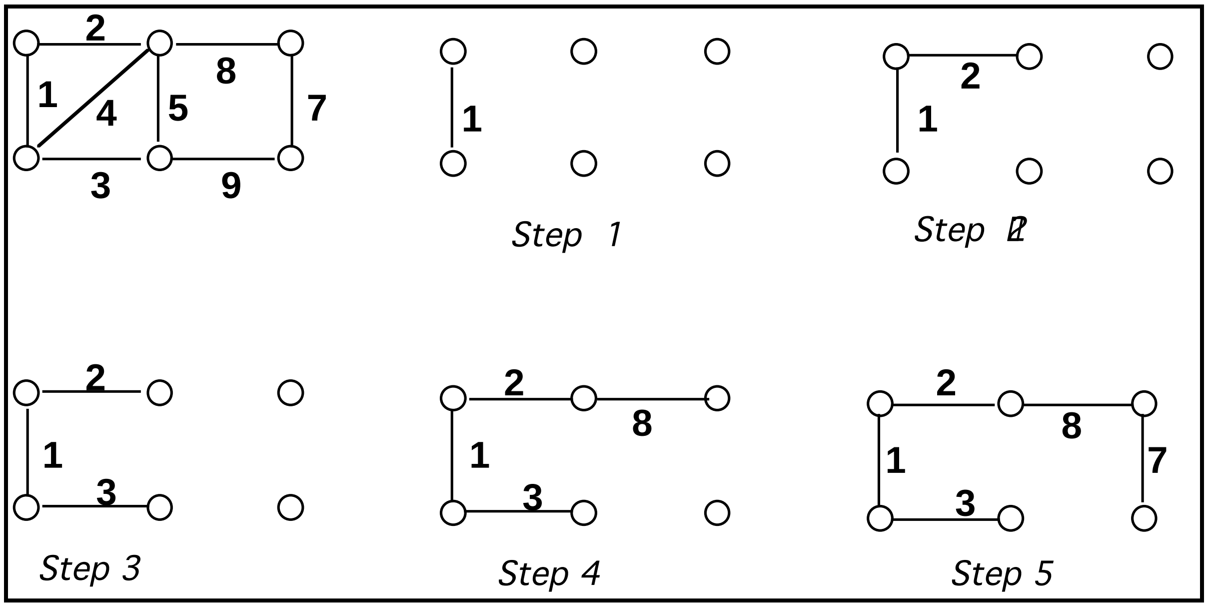
- **Is spanning tree unique?**

- **What makes for a good spanning tree?**

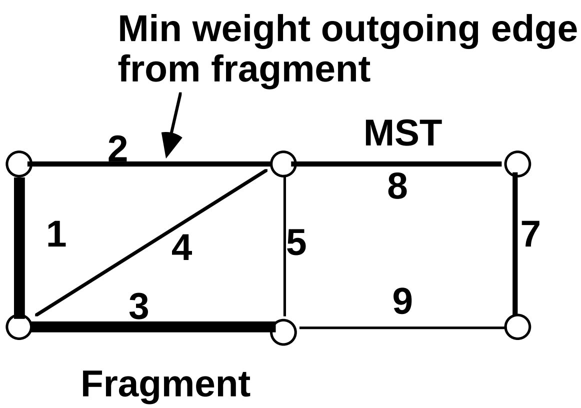
Minimum Weight Spanning Tree (MST)

- Generic MST algorithm steps:
  - Given a collection of sub-trees of an MST (called fragments) add a minimum weight outgoing edge to some fragment
- Prim-Dijkstra: Start with an arbitrary single node as a fragment
  - Add minimum weight outgoing edge
- Kruskal: Start with each node as a fragment;
  - Add the minimum weight outgoing edge, minimized over all fragments

Prim-Dijkstra Algorithm



Kruskal's Algorithm Example



- **Suppose the arcs of weight 1 and 3 are a fragment**
  - **Consider any spanning tree using those arcs and the arc of weight 4, say, which is an outgoing arc from the fragment**
  - **Suppose that spanning tree does not use the arc of weight 2**
  - **Removing the arc of weight 4 and adding the arc of weight 2 yields another tree of smaller weight**
  - **Thus an outgoing arc of min weight from fragment must be in MST**

Shortest Path routing

- Each link has a cost that reflects
  - The length of the link
  - Delay on the link
  - Congestion
  - $$ cost
- Cost may change with time

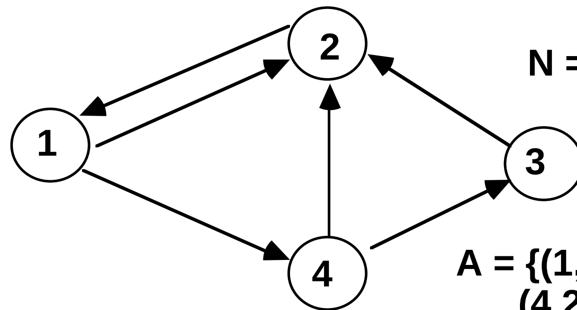
- The length of the route is the sum of the costs along the route

- The shortest path is the path with minimum length

- Shortest Path algorithms
  - Bellman-Ford: centralized and distributed versions
  - Dijkstra's algorithm
  - Many others

Directed graphs (digraphs)

- A directed graph (digraph) G = (N,A) is a finite nonempty set of nodes N and a set of ordered node pairs A called directed arcs.



N = {1,2,3,4}

A = {(1,2), (2,1),(1,4), (4,2), (4,3),(3,2)}

- Directed walk: (4,2,1,4,3,2)
- Directed path: (4,2,1)
- Directed cycle: (4,2,1,4)
- Data networks are best represented with digraphs, although typically links tend to be bi-directional (cost may differ in each direction)
  - For simplicity we will use bi-directional links of equal costs in our examples

Dijkstra's algorithm

- Find the shortest path from a given source node to all other nodes
  - Requires non-negative arc weights
- Algorithm works in stages:
  - Stage k: the k closest nodes to the source have been found
  - Stage k+1: Given k closest nodes to the source node, find k+1st
- Key observation: the path to the k+1st closest nodes includes only nodes from among the k closest nodes
- Let M be the set of nodes already incorporated by the algorithm
  - Start with $D_n = d_{sn}$ for all n (D_n = shortest path distance from node n to the source node
  - Repeat until M=N

    Find node $w \notin M$ which has the next least cost distance to the source node

    Add w to M

    Update distances: $D_n = \min [D_n, D_w + d_{wn}]$ (for all nodes $n \notin M$)
  - Notice that the update of D_n need only be done for nodes not already in M and that the update only requires the computation of a new distance by going through the newly added node w

Dijkstra example

Dijkstra's algorithm implementation

- Centralized version: Single node gets topology information and computes the routes
  - Routes can then be broadcast to the rest of the network
- Distributed version: each node i broadcasts {dij all j} to all nodes of the network; all nodes can then calculate shortest paths to each other node
  - Open Shortest Path First (OSPF) protocol used in the internet

Bellman Ford algorithm

- Finds the shortest paths, from a given source node, say node 1, to all other nodes.

- General idea:

  – First find the shortest single arc path,
  – Then the shortest path of at most two arcs, etc.
  – Let $d_{ij}=\infty$ if (i,j) is not an arc.

- Let $D_i(h)$ be the shortest distance from 1 to i using at most h arcs.
  – $D_i(1) = d_{1i}$; $i \neq 1$ $D_1(1) = 0$
  – $D_i(h+1) = \min_{\{j\}} [D_j(h) + d_{ji}]$; $i \neq 1$ $D_1(h+1) = 0$

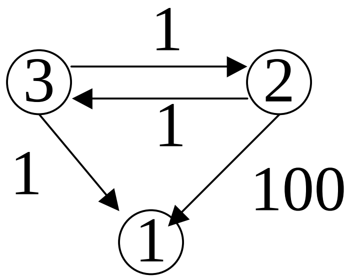
- If all weights are positive, algorithm terminates in N-1 steps.

Bellman Ford - example

Distributed Bellman Ford

- Link costs may change over time
  - Changes in traffic conditions
  - Link failures
  - Mobility
- Each node maintains its own routing table
  - Need to update table regularly to reflect changes in network
- Let Di be the shortest distance from node i to the destination
  - Di = min {j} [Dj + dij] : update equation
- Each node (i) regularly updates the values of Di using the update equation
  - Each node maintains the values of dij to its neighbors, as well as values of Dj received from its neighbors
  - Uses those to compute Di and send new value of Di to its neighbors
  - If no changes occur in the network, algorithm will converge to shortest paths in no more than N steps

Slow reaction to link failures



- **Start with D3=1 and D2=100**
  - **After one iteration node 2 receives D3=1 and D2 = min [1+1, 100] = 2**
- **In practice, link lengths occasionally change**
  - **Suppose link between 3 and 1fails (I.e., d31=infinity)**
  - **Node 3 will update D3 = d32 + D2 = 3**
  - **In the next step node 2 will update: D2 = d23+D3 = 4**
  - **It will take nearly 100 iterations before node 2 converges on the correct route to node 1**
- **Possible solutions:**
  - **Propagate route information as well**
  - **Wait before rerouting along a path with increasing cost**
    - **Node next to failed link should announce D=infinity for some time to prevent loops**

Routing in the Internet

- **Autonomous systems (AS)**
  - **Internet is divided into AS's each under the control of a single authority**
- **Routing protocol can be classified in two categories**
  - **Interior protocols - operate within an AS**
  - **Exterior protocols - operate between AS's**

- **Interior protocols**
  - **Typically use shortest path algorithms**
    - **Distance vector - based on distributed Bellman-ford**
    - **link state protocols - Based on "distributed" Dijkstra's**

Distance vector protocols

- **Based on distributed Bellman-Ford**
  - **Nodes exchange routing table information with their neighbors**

- **Examples:**
  - **Routing information protocols (RIP)**
    - **Metric used is hop-count (dij=1)**
    - **Routing information exchanged every 30 seconds**

  - **Interior Gateway Routing Protocol (IGRP)**
    - **CISCO proprietary**
    - **Metric takes load into account**
    - **Dij ~ $1/(\mu-\lambda)$ (estimate delay through link)**

    - **Update every 90 seconds**
    - **Multi-path routing capability**

Link State Protocols

- **Based on Dijkstra's Shortest path algorithm**
  - **Avoids loops**
  - **Routers monitor the state of their outgoing links**
  - **Routers broadcast the state of their links within the AS**
  - **Every node knows the status of all links and can calculate all routes using dijkstra's algorithm**
    - **Nonetheless, nodes only send packet to the next node along the route with the packets destination address. The next node will look-up the address in the routing table**
- **Example: Open Shortest Path First (OSPF) commonly used in the internet**
- **Link State protocols typically generate less "control" traffic than Distance-vector**

Inter-Domain routing

- Used to route packets across different AS's

- Options:

  – Static routing - manually configured routes

  – Distance-vector routing

    Exterior Gateway Protocol (EGP)

    Border Gateway Protocol (BGP)

- Issues

  – What cost "metric" to use for Distance-Vector routing

    Policy issues: Network provider A may not want B's packets routed through its network or two network providers may have an agreement

    Cost issues: Network providers may charge each other for delivery of packets