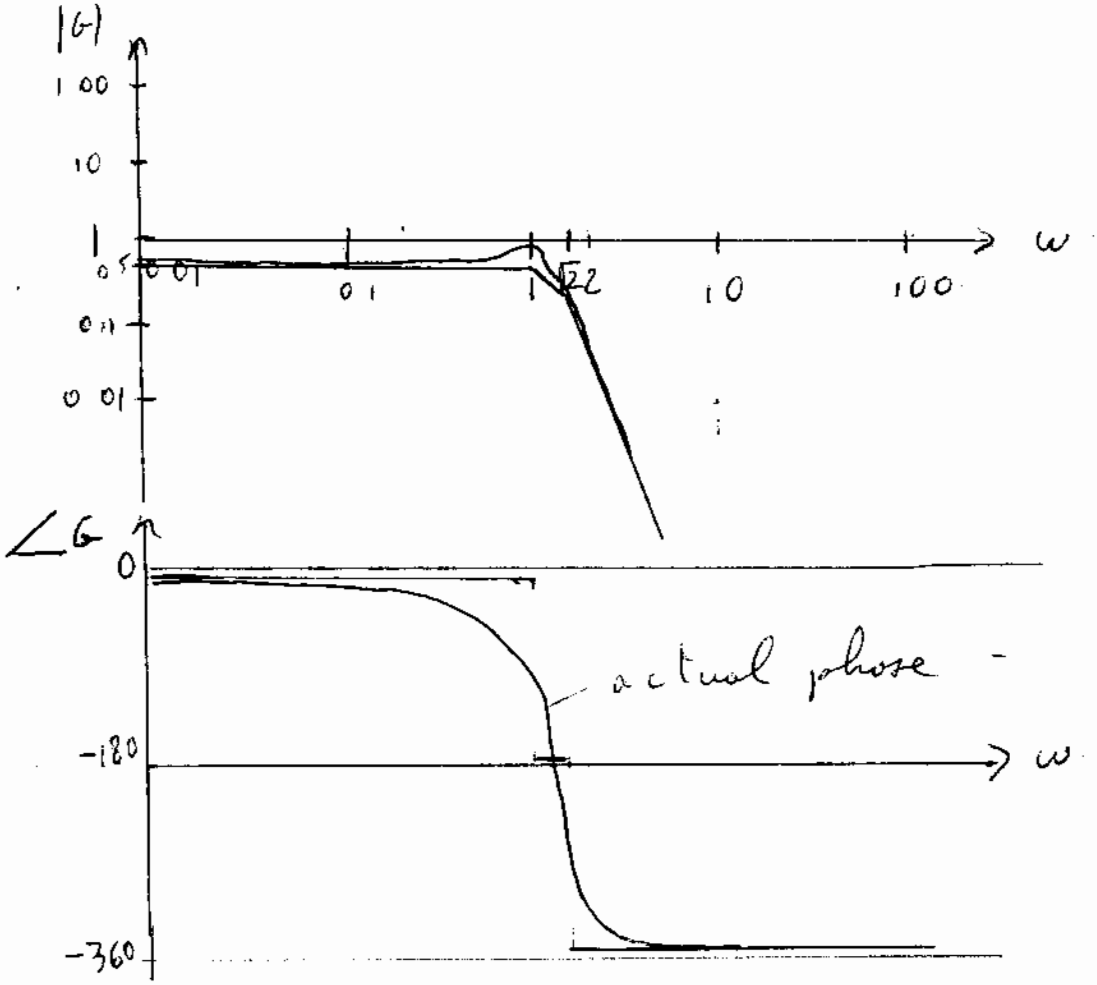
First HW solutions

1] a) $G(s) = \dfrac{1}{(s+1)^2(s^2+s+2)} = \dfrac{1}{2}\dfrac{1}{(s+1)^2\left(\left(\frac{s}{\sqrt{2}}\right)^2+\frac{s}{2}+1\right)}$

$= \dfrac{1}{2}\dfrac{1}{(s+1)^2\left(\left(\frac{s}{\sqrt{2}}\right)^2+2\frac{1}{2\sqrt{2}}\left(\frac{s}{\sqrt{2}}\right)+1\right)} \Rightarrow$

important breakpoints: $\omega = 1$ $\omega = \sqrt{2}$

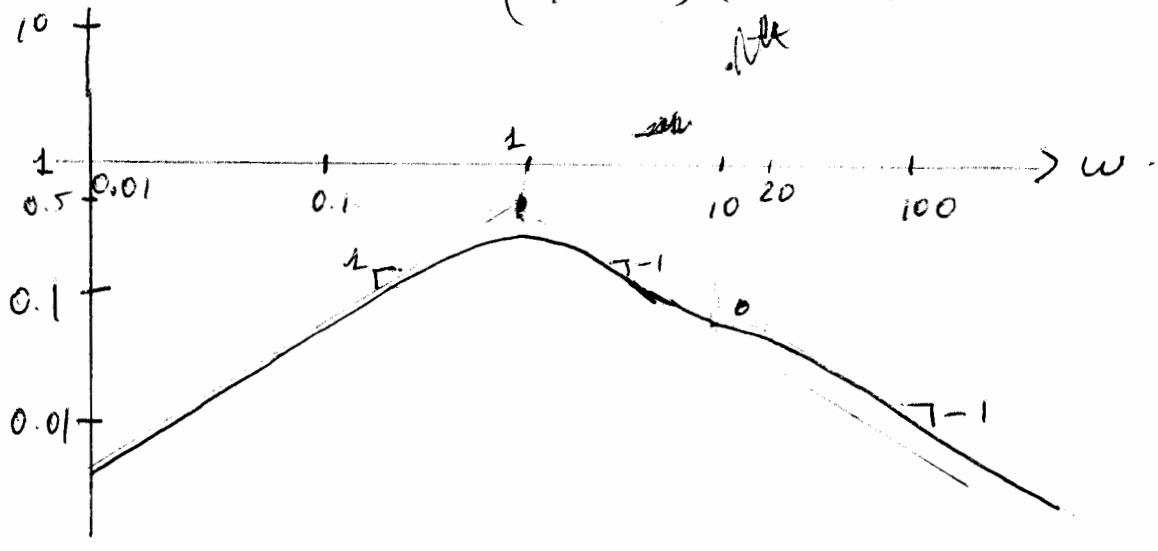


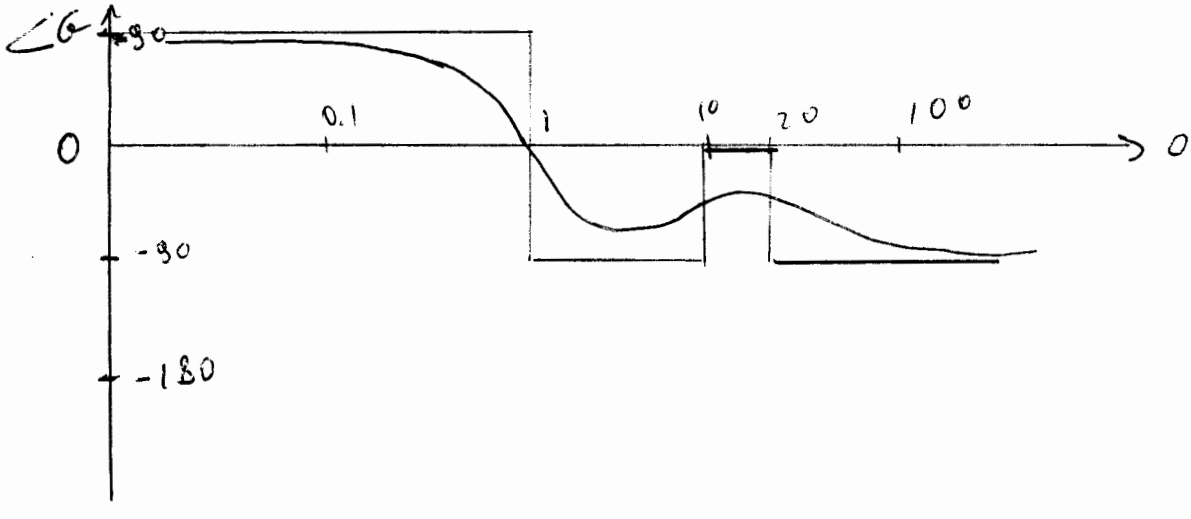
b) $G(s) = \dfrac{4s(s+10)}{(s+20)(4s^2+5s+4)}$

$= \dfrac{4 \times 10}{20 \times 4} \times \dfrac{s(s/10+1)}{(s/20+1)(s^2+\frac{5}{4}s+1)}$

$= \dfrac{1}{2} \dfrac{s(s/10+1)}{(s/20+1)(s^2+2\frac{5}{8}s+1)}$

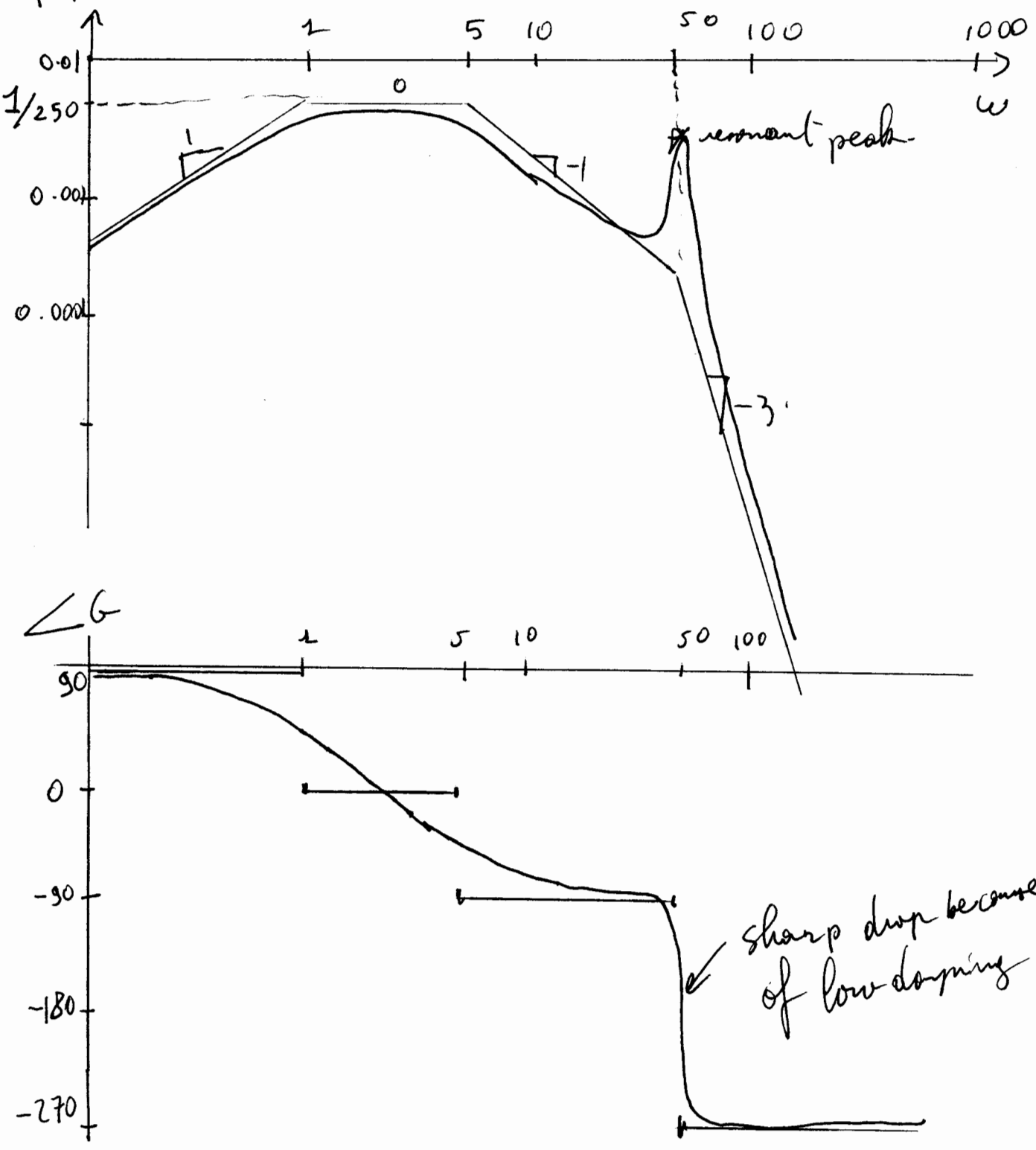
|G|





c) $G(s) = \dfrac{s}{(s+1)(s+5)(s^2+5s+(50)^2)}$

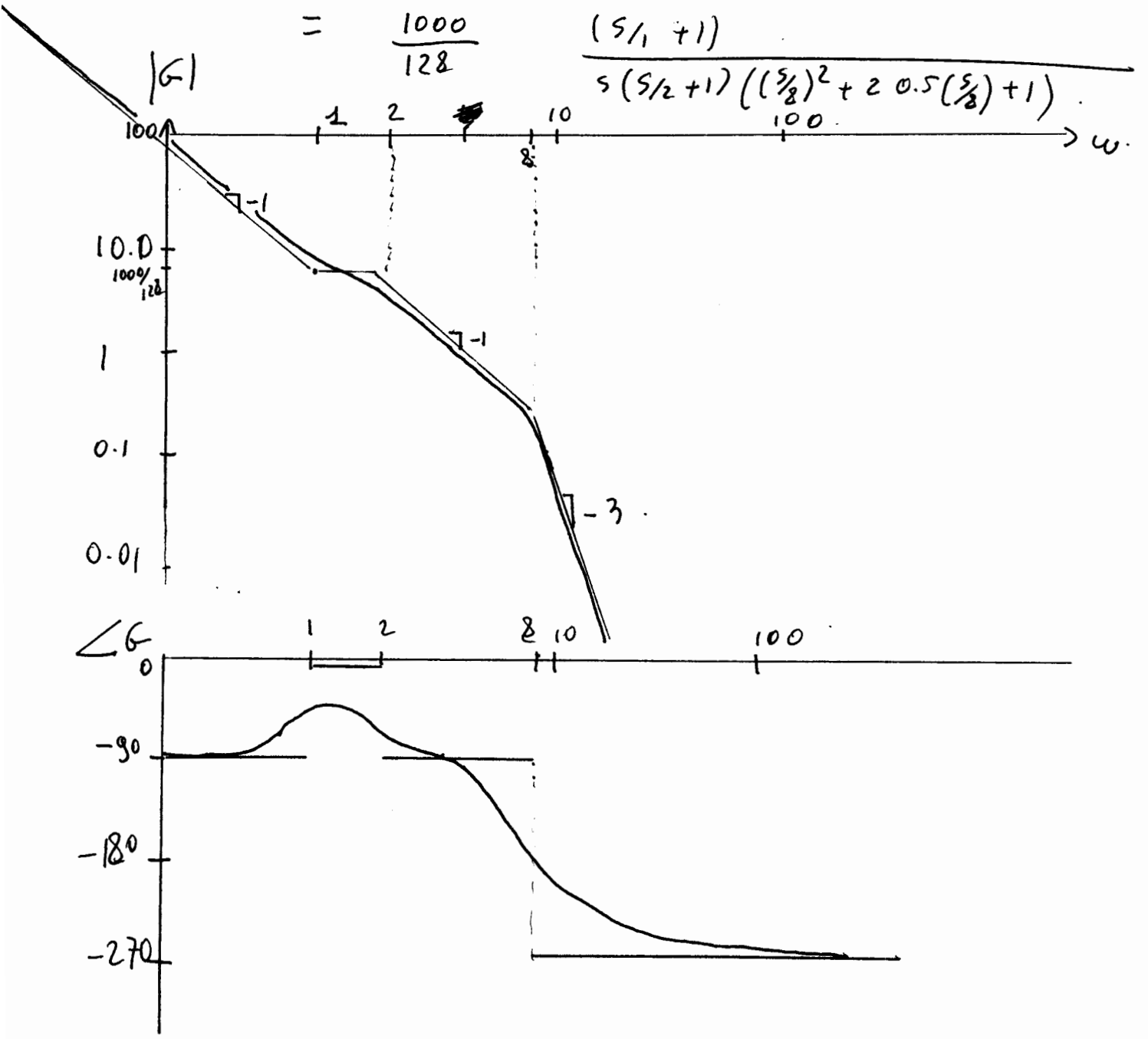
$= \dfrac{s}{5\times 50\,(s/1+1)(s/5+1)\left(\left(\frac{s}{50}\right)^2+2\left(\frac{1}{20}\right)\frac{s}{50}+1\right)}$



d] $G(s) = \dfrac{1000\ (s+1)}{s(s+2)(s^2+8s+64)}$.

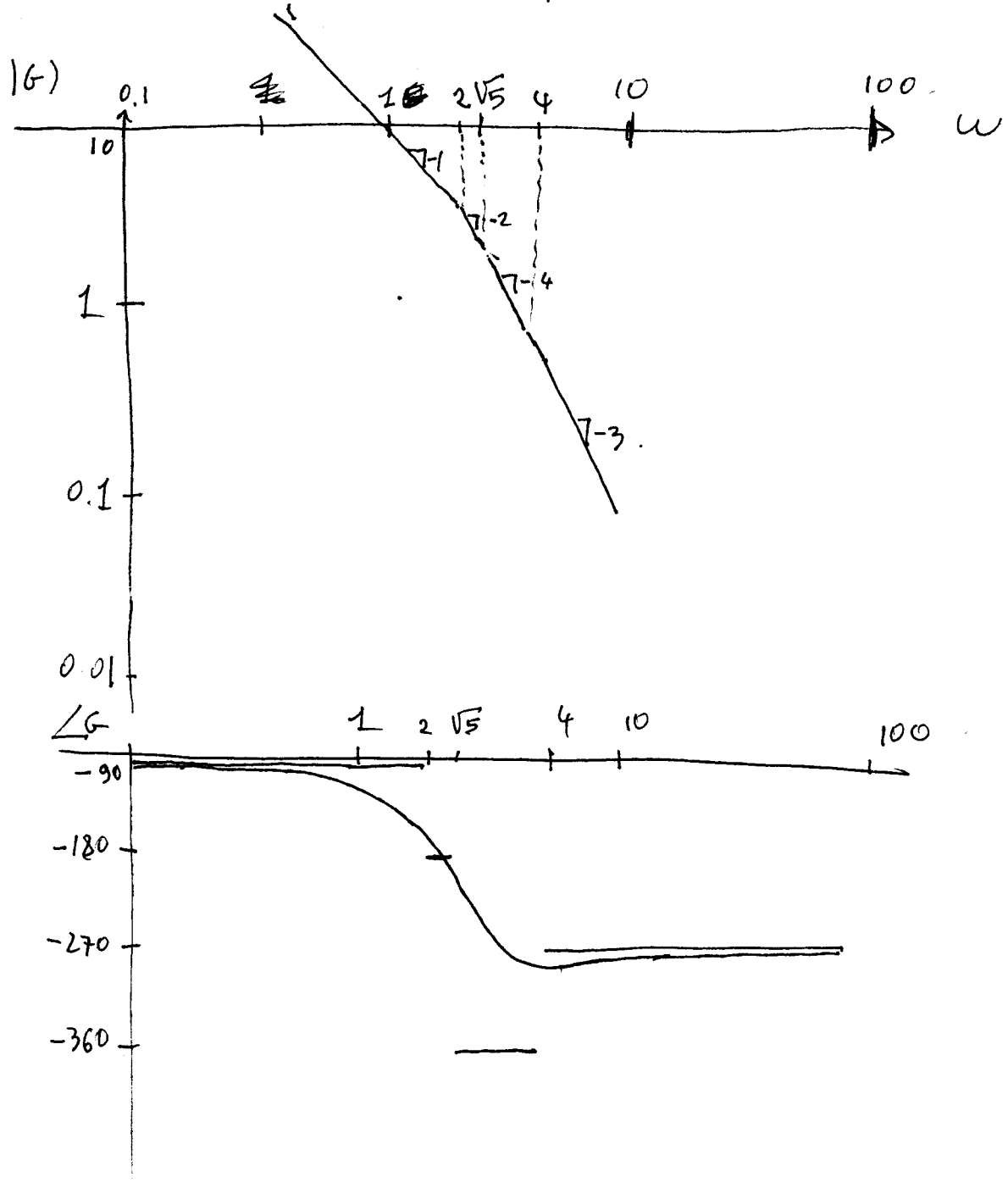
$= \dfrac{1000}{2\times 64} \dfrac{(s/1+1)}{s(s/2+1)\left(\left(\frac{s}{8}\right)^2+2\frac{1}{2}\left(\frac{s}{8}\right)+1\right)}$

$= \dfrac{1000}{128} \dfrac{(s/1+1)}{s(s/2+1)\left(\left(\frac{s}{8}\right)^2+2\,0.5\left(\frac{s}{8}\right)+1\right)}$.

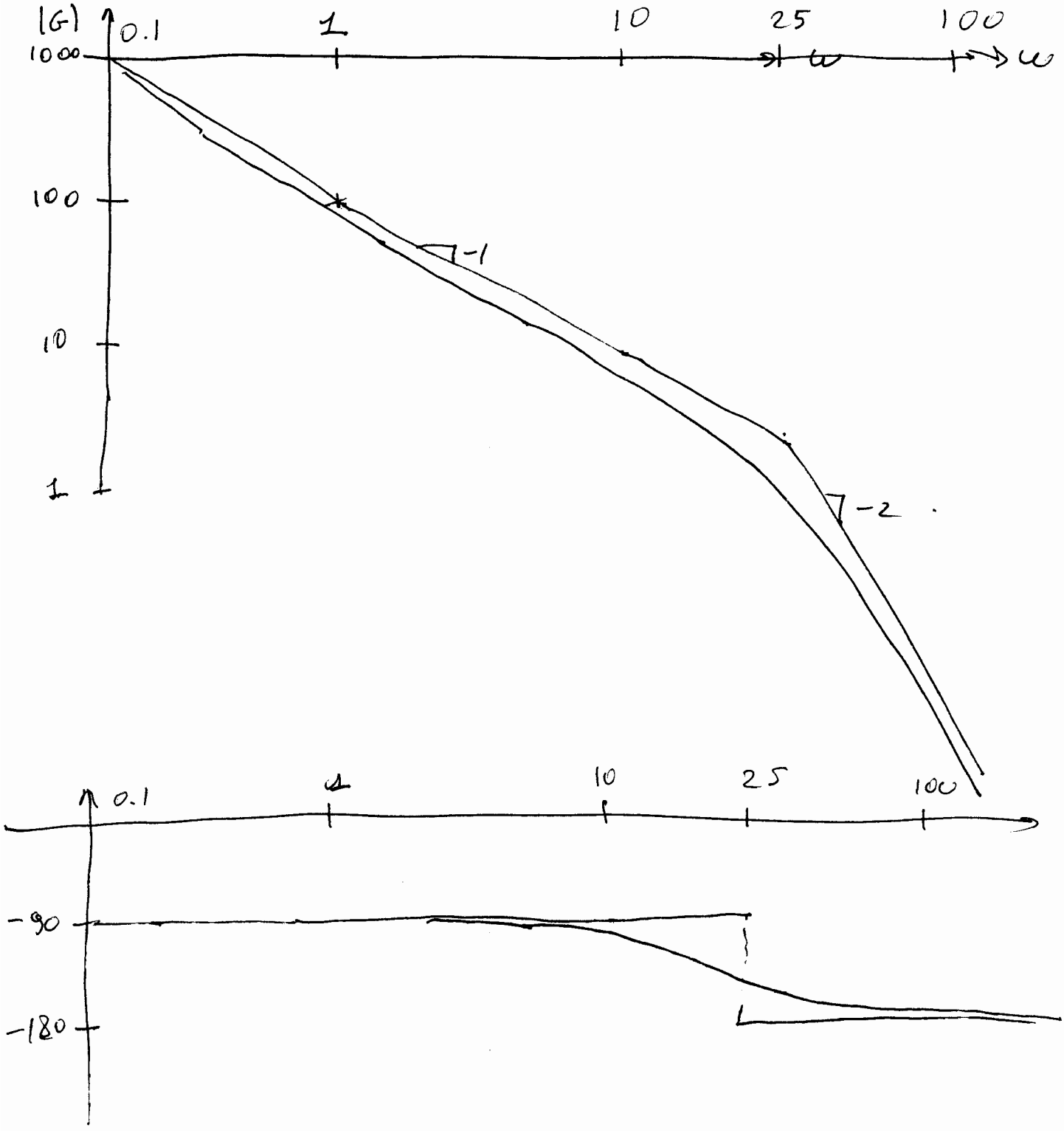


$$G(s) = \frac{10(s+4)}{s(s+2)(s^2+2s+5)} = \frac{10\times 4\,(s/4+1)}{2\times 5\times s(s/2+1)\left(\left(\frac{s}{\sqrt{5}}\right)^2+2\frac{1}{\sqrt{5}}\frac{s}{\sqrt{5}}+1\right)}$$

$$= \frac{10\,(s/4+1)}{s(s/2+1)\left(\left(\frac{s}{\sqrt{5}}\right)^2+2\,\frac{1}{\sqrt{5}}\left(\frac{s}{\sqrt{5}}\right)+1\right)}$$



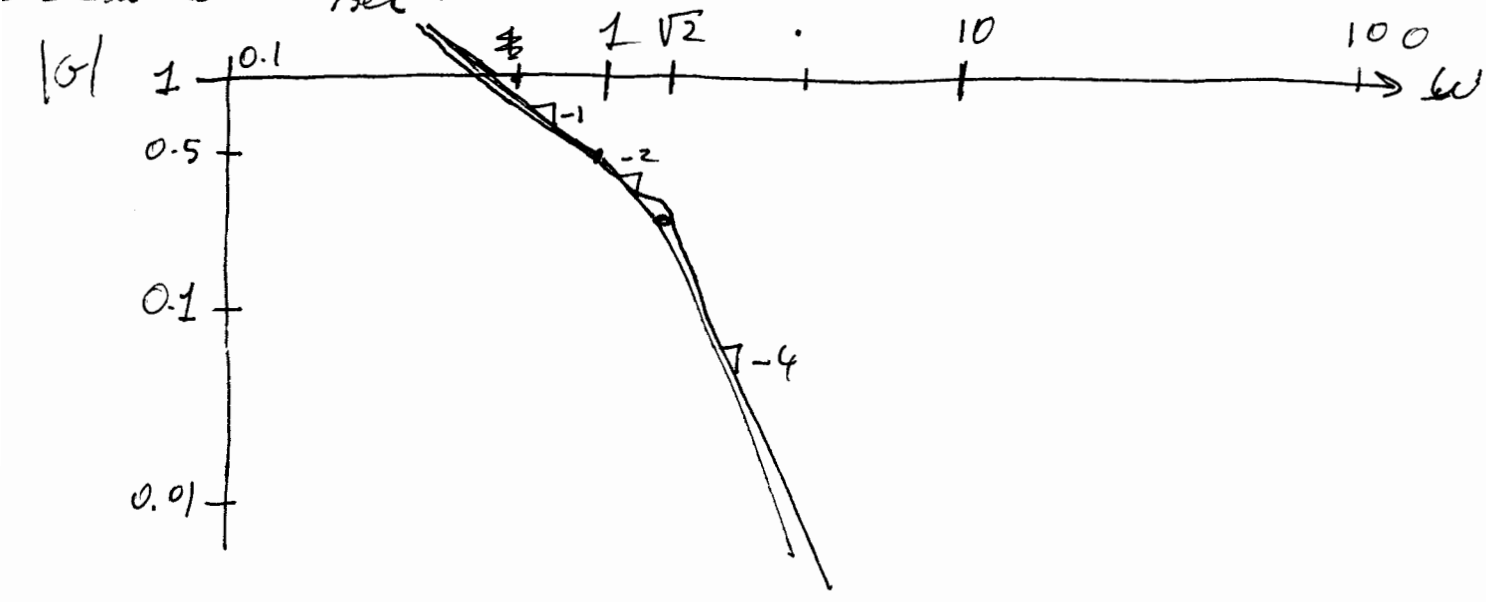
$$G(s) = \frac{2500}{s(s+25)} = \frac{100}{s\left(\frac{s}{25}+1\right)}$$

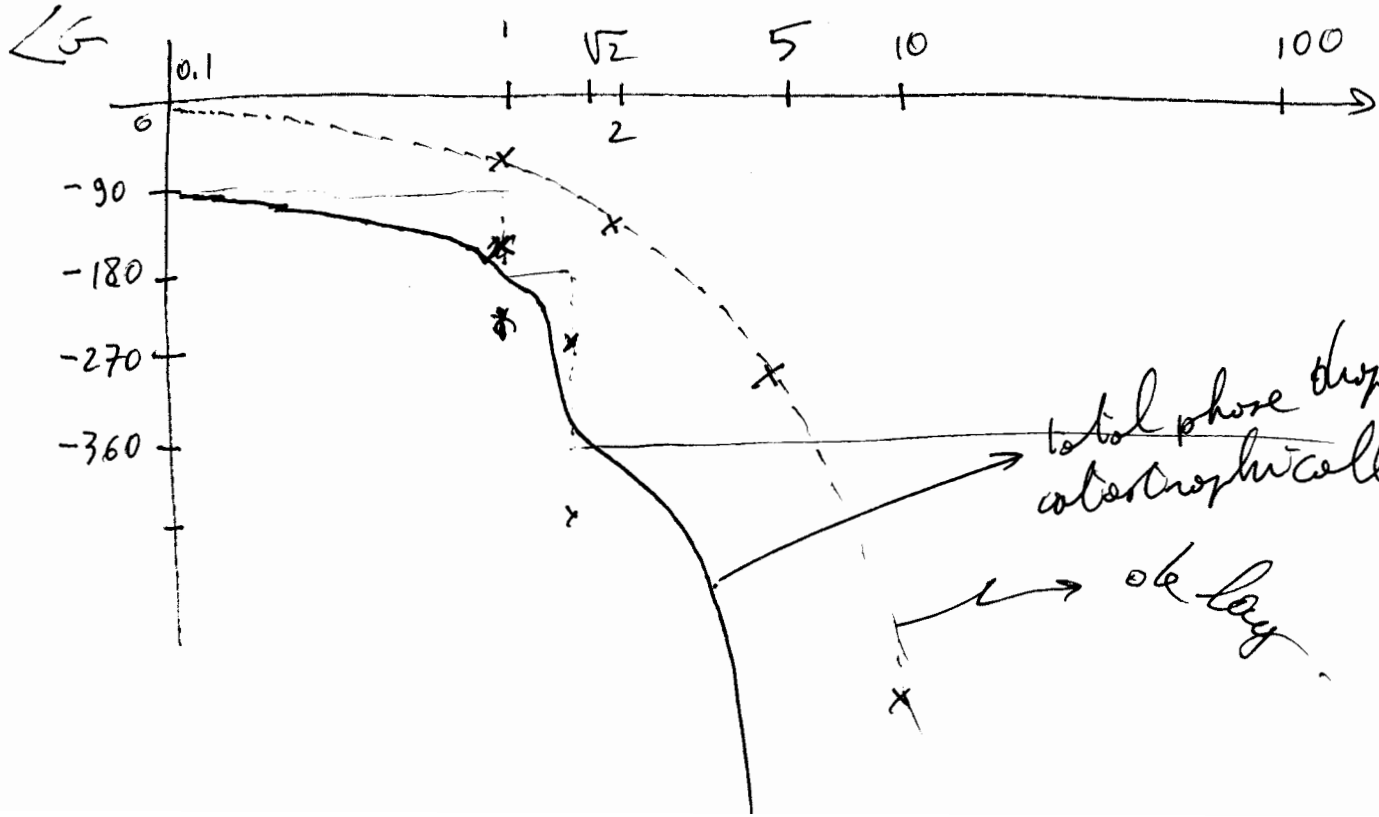


(g) $G(s) = \dfrac{e^{-s}}{s(1+s)(s^2+s+2)}$

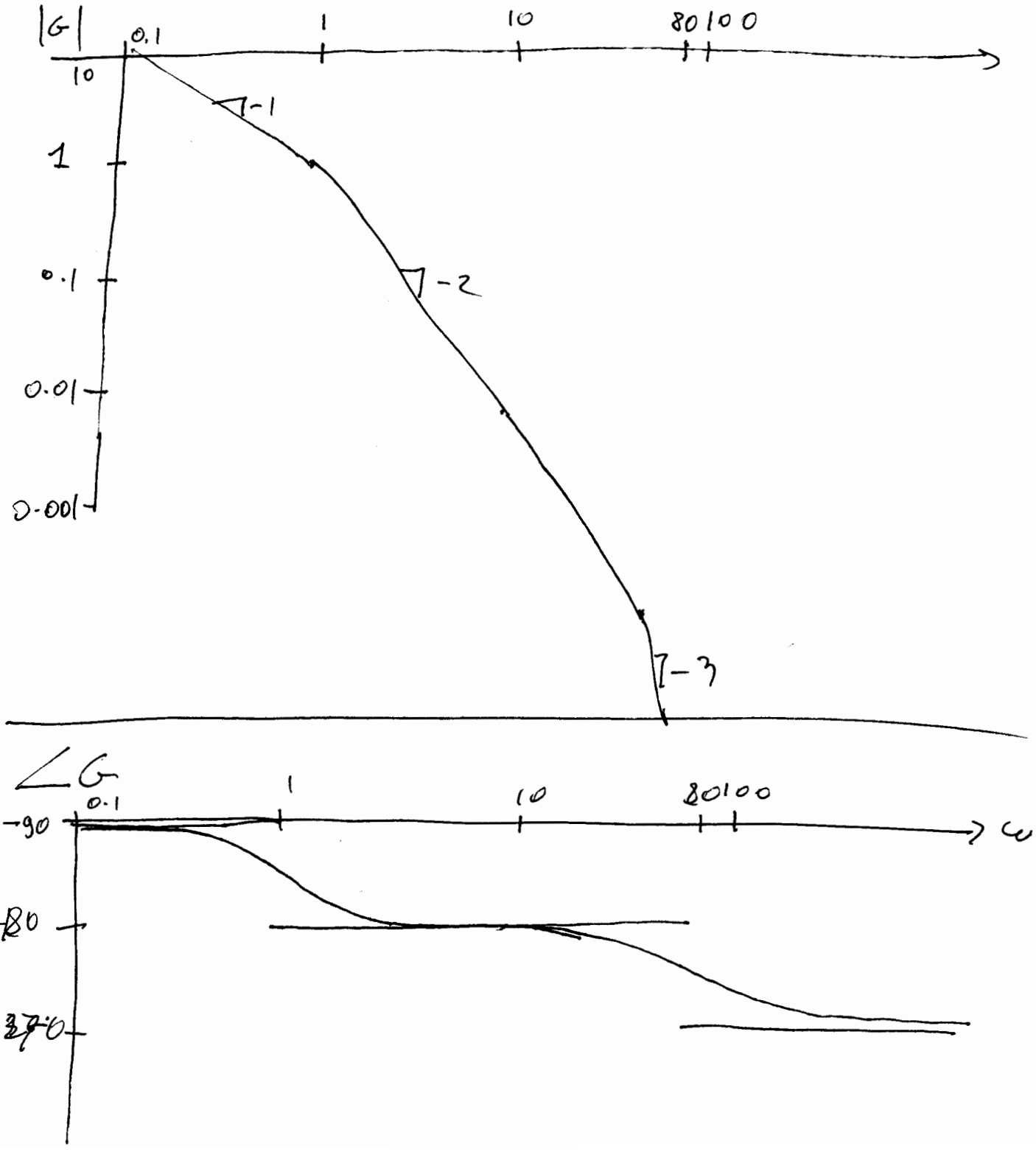
$= \dfrac{e^{-s}}{2s(s_1+1)\left(\left(\frac{s}{\sqrt{2}}\right)^2 + \frac{2}{2\sqrt{2}}\left(\frac{s}{\sqrt{2}}\right) + 1\right)}$ → time delay of 1 sec.

e^{-s} = 1 rad @ 1 rad/sec.

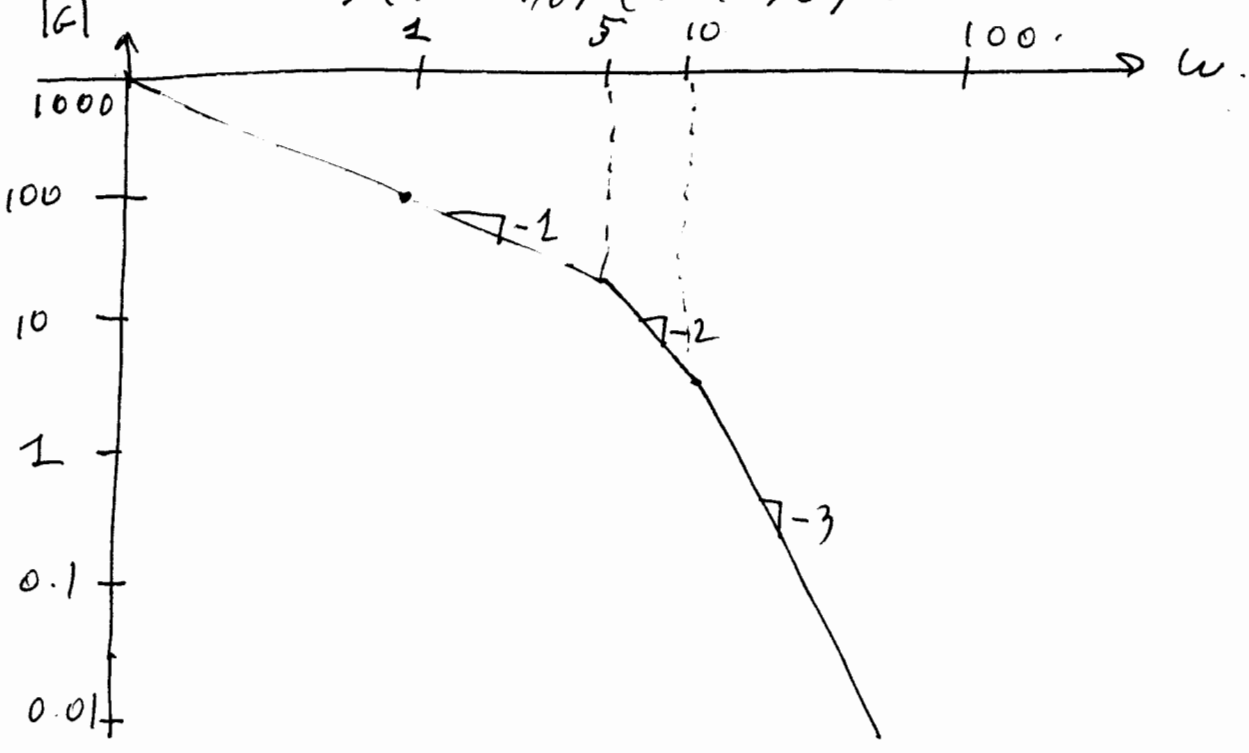


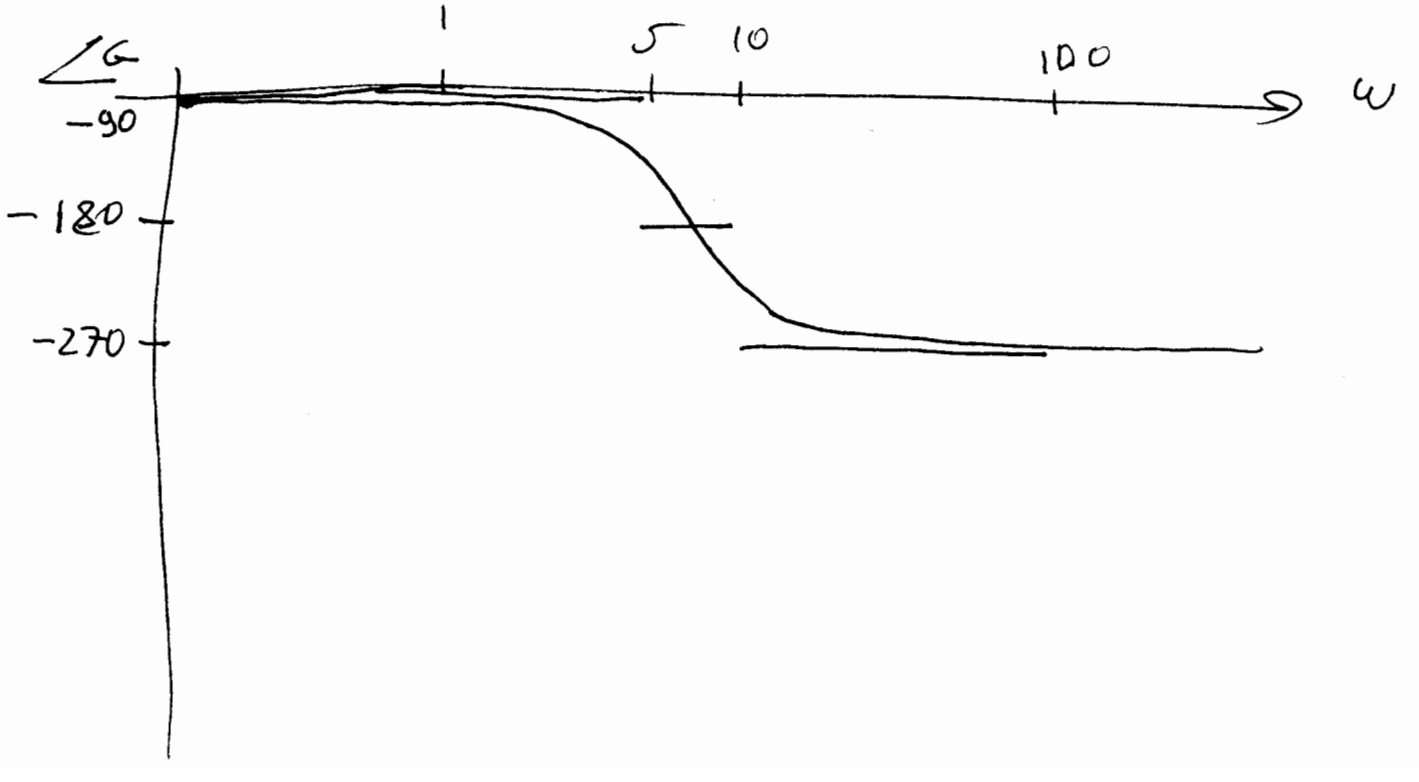


(h) $G(s) = \dfrac{1}{s(1+s)(1+s/80)}$.



(i) $G(s) = \dfrac{100}{s(1+s/10)(1+s/5)}$.

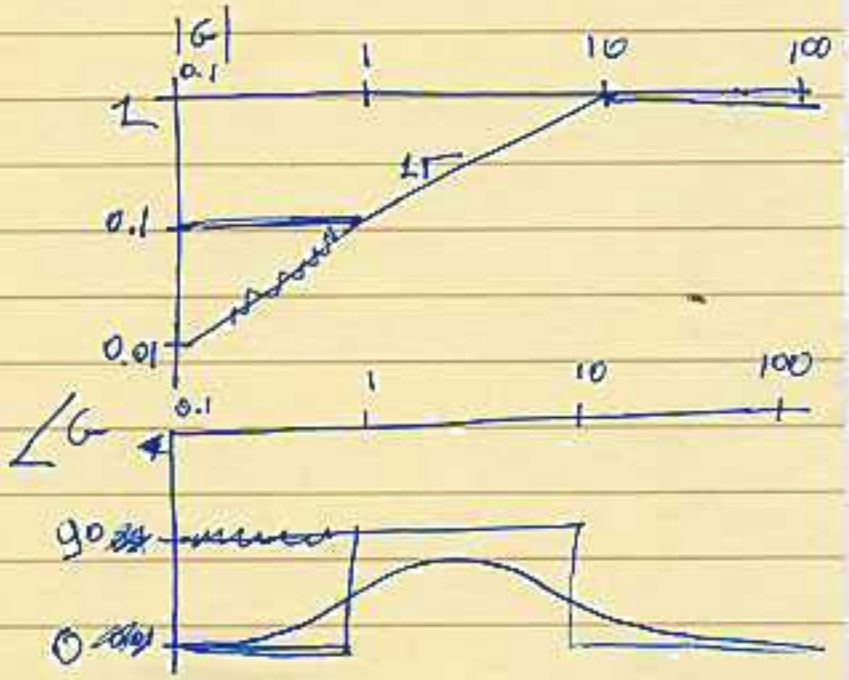


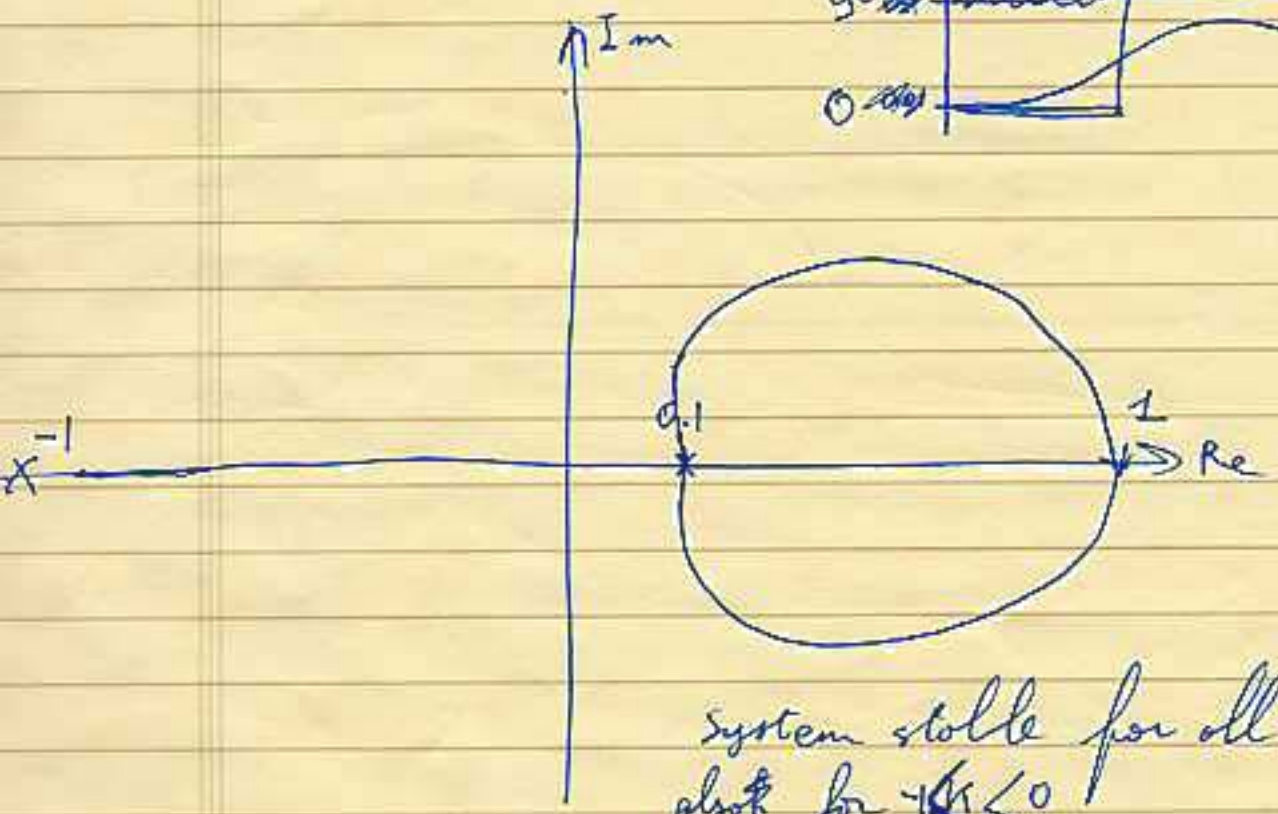


2] a] $KG(s) = \dfrac{K(s+1)}{(s+10)}$

Begin with Bode Plot.

$G(s) = \dfrac{(s+1)}{10(s/10+1)}$



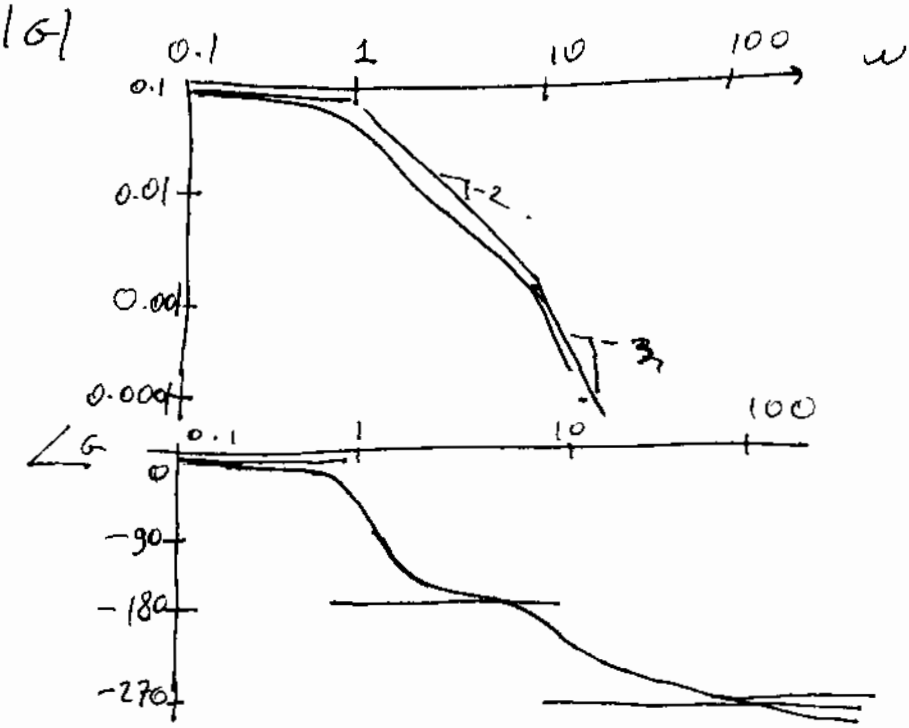


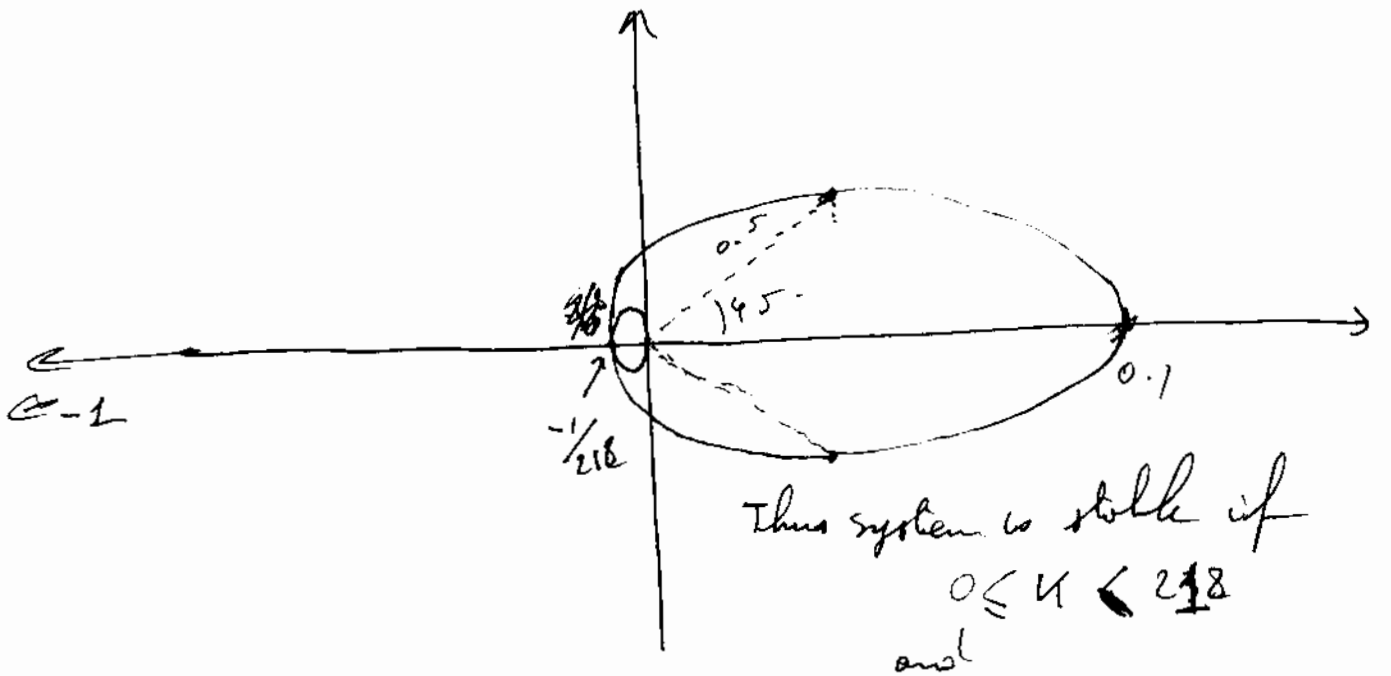
System stable for all $K > 0$
also for $-10 \leq K < 0$

and $K < -10$.

b) $KG(s) = \dfrac{K}{(s+10)(s+1)^2}$.

$G(s) = \dfrac{1}{10(s/10+1)(s+1)^2}$.





Thus system is stable if

$0 \leq K < 218$

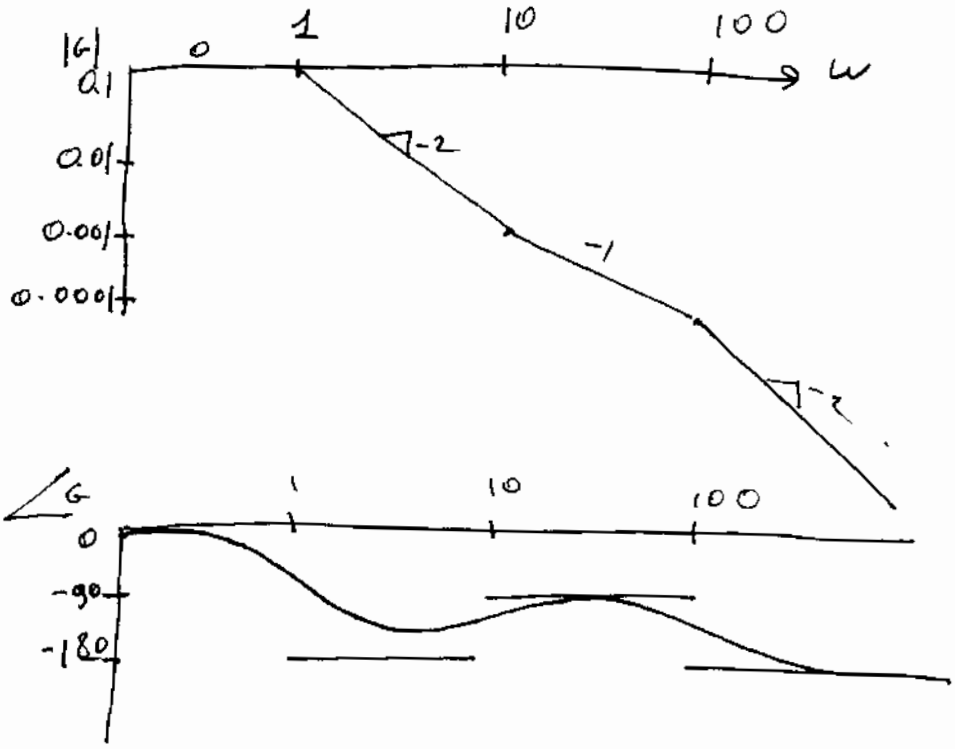
and

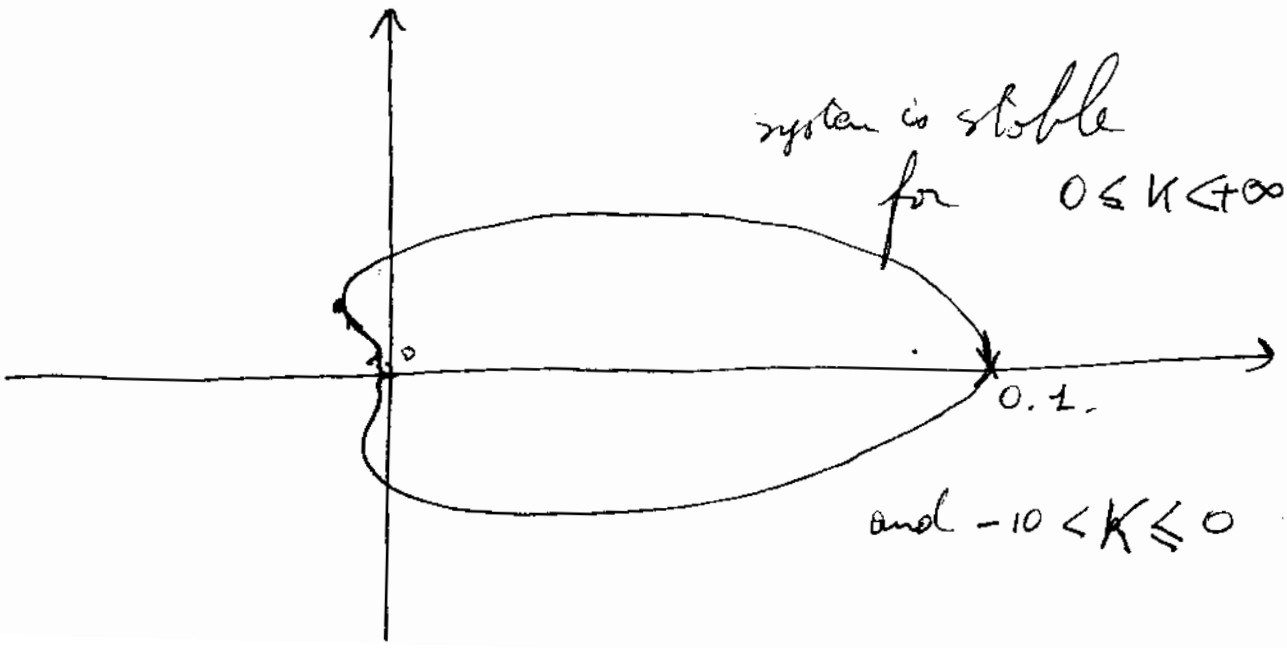
$-10 < K \leq 0$

$$KG(s) = \frac{K(s+10)(s+1)}{(s+100)(s+1)^3}$$

$$= \frac{K(s+10)}{(s+100)(s+1)^2} = \frac{K}{10}\,\frac{(s/10+1)}{(s/100+1)(s+1)^2}$$

Bode Plot:





system is stable for $0 \le K < +\infty$

and $-10 < K \le 0$