Lecture 34 Dec 5, 03 ①

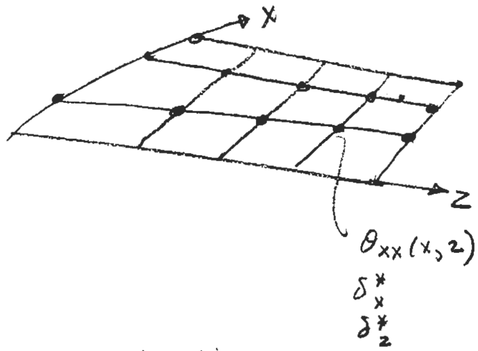
3D Boundary Layers

> 3D Integral Method Soln Proc

B> B-Cs, Well-posedness

c> 3D Interaction

A> <u>Solution Technique</u>



- March solution in x-direction
  - u_e, w_e prescribed
- x-line like treatment
- Fundamental unknowns: θ_{xx}, δ^*_x, δ^*_z – all others emp. related

x, z-mom + K.E eqns give

$$\Rightarrow \quad \bar{\bar{A}}\frac{\partial \vec{f}}{\partial x} + \bar{\bar{B}}\frac{\partial \vec{f}}{\partial z} = \vec{g} \qquad ; \qquad \vec{f} = \text{unknowns.}$$

$\bar{A}, \bar{B}, \vec{g}$ depend on local solution

We need $\frac{\partial \vec{f}}{\partial x}$ explicitly

$$\therefore \quad \frac{\partial \vec{f}}{\partial x} + \bar{\bar{C}}\frac{\partial \vec{f}}{\partial z} = \vec{h} \qquad C = A^{-1}B \;,\; h = A^{-1}g$$

$\bar{\bar{A}}$ is singular if $\delta^*_x/\theta_{xx} \simeq 3$ (2D like sep.)

→ Cannot integrate past sep. line

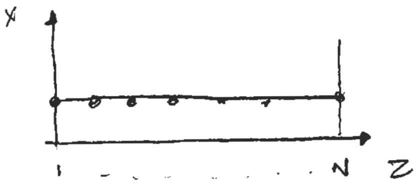
$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\ \tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\sqrt{M^2-1}\right) - \tan^{-1}\sqrt{M^2-1}$$

$$S_1 = V/u \qquad \beta = \tan^{-1}(V/u)$$

$$\nu(M) \pm \beta = \nu(\hat{M}_1) \pm \tan^{-1}(\hat{S}_1)$$

Assume no separation

Need 3 BCs on each $x =$ const. line



Implicit system along x-line

3N unknowns

$3(N-1)$ eqns

3 BCs – linear comb. of f_1, f_2, f_3.

Analogous to 2D F.D scheme.

3N unknowns F_j, U_j, S_j

$3(N-1)$ at ξ

3 BCs.

Define

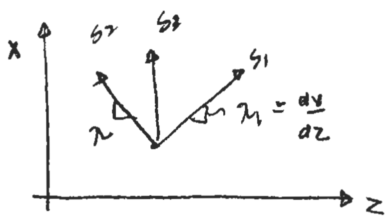
$$\bar{\bar{T}} \text{ s.t. } \quad \bar{\bar{T}}^{-1}\bar{\bar{C}}\,\bar{\bar{T}} = \left[\ \lambda_{1,2,3}\ \right]$$

$$T^{-1}\frac{\partial \vec{f}}{\partial x} + (T^{-1}CT)\,T^{-1}\frac{\partial f}{\partial z} = T^{-1}h$$

$$\Rightarrow \quad \frac{\partial f'}{\partial x} + \Lambda\frac{\partial f'}{\partial z} = h' \qquad f' = T^{-1}f$$

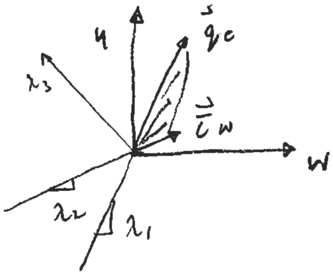
charac eqn. uncoupled

$\vec{f}'$ are some linear combo of $\theta_{xx}, \delta^*_x, \delta^*_z$



In practice, 2 of the λs are along $\vec{q}_e$, wall stream line

3rd λ is some other direction



Well Posedness

PDE system

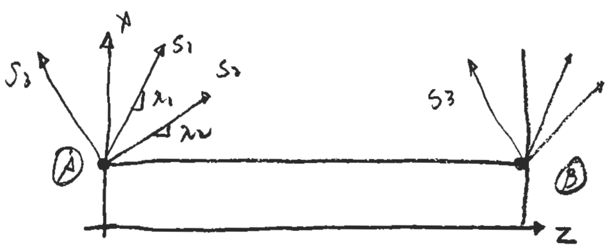
$$\vec f = \{\theta_{xo}, \delta_x^*, \delta_z^*\}$$

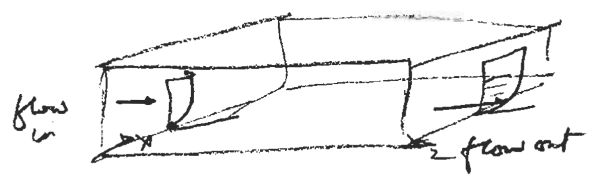
$$\frac{\partial \vec f}{\partial x} + \bar{\bar{C}} \frac{\partial \vec f}{\partial x} = \vec h$$

To allow marching, $\bar{\bar{C}}$ must have real eigenvalues λ_{1-3}

Signs define B.C

Imposing B.Cs

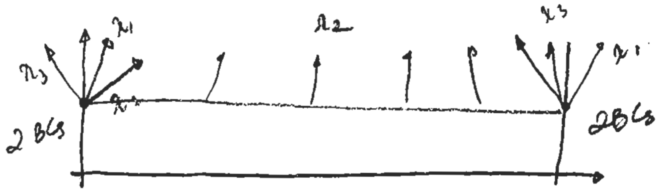


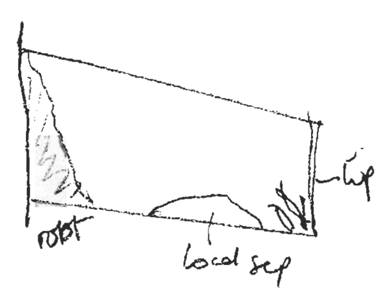


Can only impose f'_1, f'_2 at A

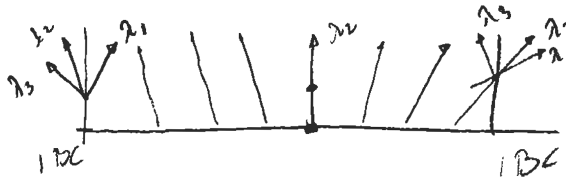
f'_3 at B

Difficulties





- Throw out one internal equation



→ Need one internal BC

→ f^2 extrapolated where $\lambda \to \infty$
$d\delta/dz = 0$

Implication → Flow dependant marching complicated !

Coordinate System

Rotationally invariant in y.

3 possibilities :

orthogonal – { ① x , y , z — cartesian — arb
② s , y , n — streamline aligned — unique
$s \parallel \vec{q_e}$, $n \perp \vec{q_e}$ ~~arb~~

not nec orthogonal – ③ ξ , y , η — computational — arb

– ③ is most convenient.

– ② physically relevant – comp. inconvenient

– can get ③ from CAD system – rect patches.

C) 3D IBLT