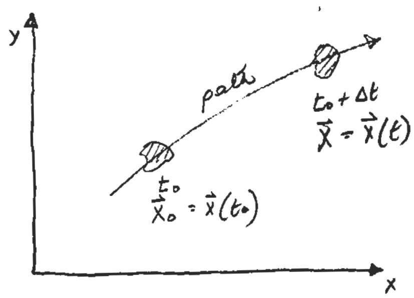
Recap of Lecture 1 – Kinematic components – convection + vorticity + strain

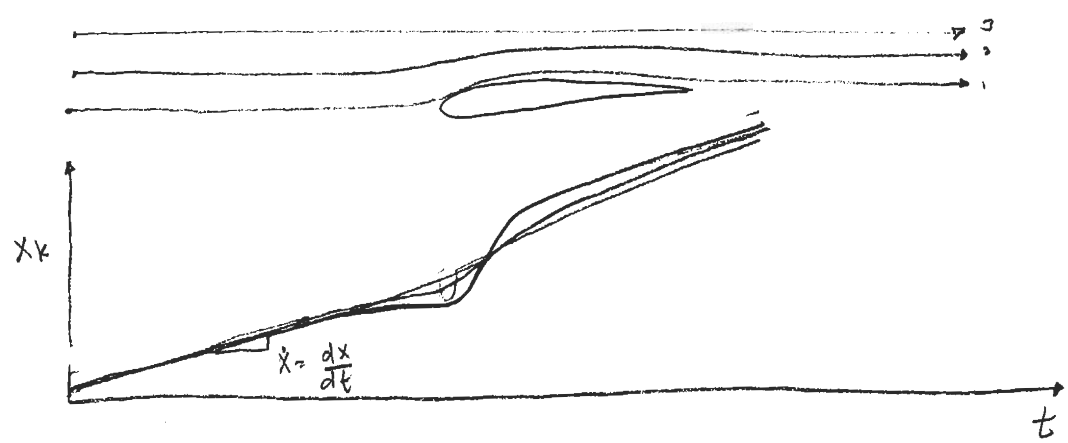
1.2 A) Lagrangian vs. Eulerian Description

Following / tracking the path of a fluid particle over time is termed the Lagrangian approach



- always unsteady.
- analysis / computation cumbersome

Example



$$\vec{a} = \left.\frac{\partial \vec{u}}{\partial t}\right|_{\vec{x}} + \frac{\partial \vec{u}}{\partial x_i} u_i$$

$$\equiv \frac{D\vec{u}}{Dt} \qquad \left(= \left.\frac{\partial \vec{u}}{\partial t}\right|_{\vec{\xi}} \right)$$

substantial derivative

Inst. $\xi_i = x_i$

so $\left.\frac{\partial u_i}{\partial \xi_j}\right|_t = \left.\frac{\partial u_i}{\partial x_j}\right|_t$

but $\left.\frac{\partial u_i}{\partial t}\right|_{\vec{\xi}} \neq \left.\frac{\partial u_i}{\partial t}\right|_{\vec{x}}$

Relates Lagrangian and Eulerian description of the flow.

In general,

$$\left.\frac{\partial (\,)}{\partial t}\right|_{\vec{\xi}_i} = \left.\frac{\partial (\,)}{\partial t}\right|_{\vec{x}} + u_i \frac{\partial (\,)}{\partial \xi_i} \equiv \frac{D(\,)}{Dt}$$

or

$$= \frac{\partial (\,)}{\partial t} + \vec{u} \cdot \nabla (\,)$$

unsteady — convective change.

If $A(t, \vec{x})$ is any field quantity, for example $\vec{u}, \rho, p, s$, etc.

$\frac{DA}{Dt} = \ldots$ RHS is called a "transport equation for A"

Note: $\frac{D(\,)}{Dt}$ has Galilean <u>invariance</u> (same value in any inertial frame of reference)

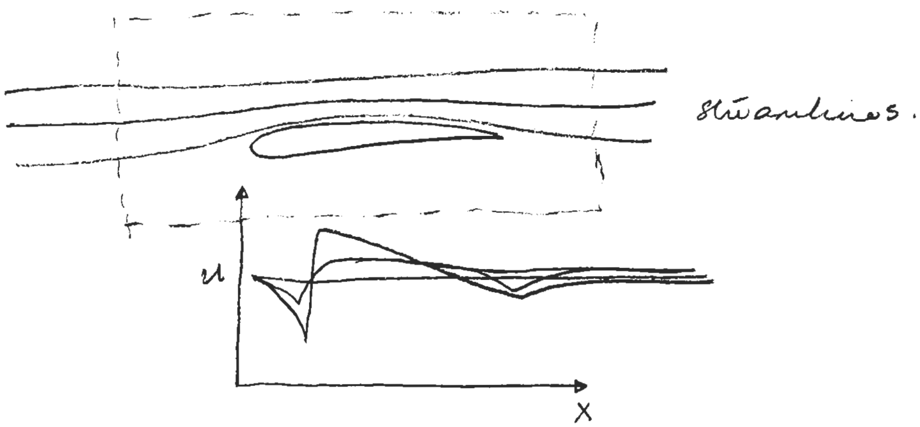
Example

$$\frac{D\vec{u}'}{Dt} = \frac{\partial \vec{u}'}{\partial t} + \vec{u}' \cdot \nabla \vec{u}' = \frac{D\vec{u}}{Dt} = \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u}$$

$$\vec{x}' = \vec{x} - \vec{c}t$$

Eulerian Description

Focusing on a control volume fixed in space, and determining the fluid behavior instantaneously in the volume is termed the Eulerian approach. We are interested in the velocity field $- \vec{u}(\vec{x},t)$, $\vec{u}(\vec{x})$ implies steady flow

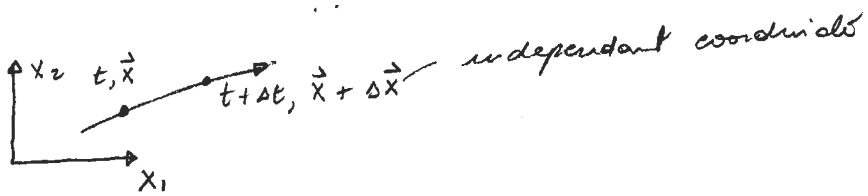


k/ Convective Relations

Absolute acceleration relative to inertial frame

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\vec{u}(t+\Delta t, \vec{\xi}) - \vec{u}(t,\vec{\xi})}{\Delta t} = \left.\frac{\partial \vec{u}}{\partial t}\right|_{\vec{\xi}} \quad \text{(Lagrangian)}$$

material coordinate system



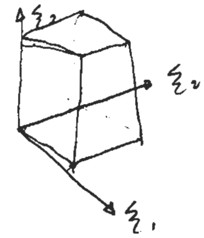
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\vec{u}(t+\Delta t, \vec{x}+\Delta\vec{x}) - \vec{u}(t,\vec{x})}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\vec{u} + \frac{\partial \vec{u}}{\partial t}\Delta t + \frac{\partial \vec{u}}{\partial x_i}\Delta x_i - \vec{u}(t,\vec{x})}{\Delta t}$$

last lecture

$\Delta x_i = u_i \Delta t$

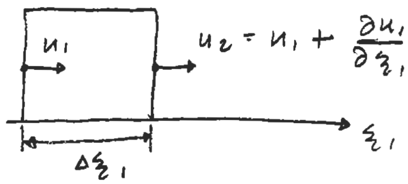
Volume rate of change



$$\Delta V = \Delta\xi_1 \cdot \Delta\xi_2 \cdot \Delta\xi_3$$

$$\ln \Delta V = \ln \Delta\xi_1 + \ln \Delta\xi_2 + \ln \Delta\xi_3$$

$$\frac{1}{\Delta V}\frac{d\,\Delta V}{dt} = \frac{1}{\Delta\xi_1}\frac{d\Delta\xi_1}{dt} + \frac{1}{\Delta\xi_2}\cdot\frac{d\Delta\xi_2}{dt} + \frac{1}{\Delta\xi_3}\cdot\frac{d\Delta\xi_3}{dt}$$



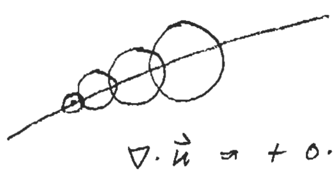
$$u_2 = u_1 + \frac{\partial u_1}{\partial \xi_1}\Delta\xi_1 + H.O.T$$

$$\therefore \quad \frac{d\,\Delta\xi_1}{dt} = u_2 - u_1 = \frac{\partial u_1}{\partial\xi_1}\cdot\Delta\xi_1 + H.O.T$$

$$\Rightarrow \quad \frac{1}{\Delta V}\frac{d\Delta V}{dt} = \frac{\partial u_1}{\partial\xi_1} + \frac{\partial u_2}{\partial\xi_2} + \frac{\partial u_3}{\partial\xi_3} + H.O.T$$

$$\frac{1}{dv}\frac{d(dv)}{dt} = \nabla\cdot\vec{u}$$ dilatation rate ($1/\text{time}$)

$(\epsilon_{11} + \epsilon_{22} + \epsilon_{33})$



$\nabla\cdot\vec{u} = +0.01/s \Rightarrow$ small volume v grows at $1\%/s$

Next lect, use in deriving conservation of mass.