AGS. J. ... May 1980

e^n Method.

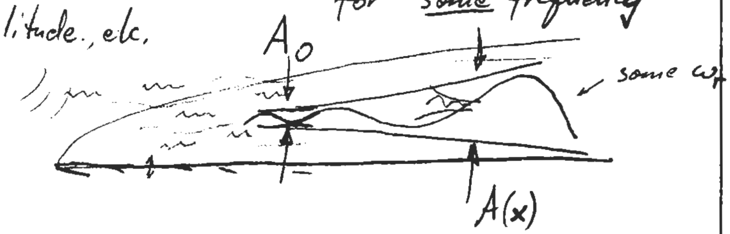
Uses Orr-Sommerfeld Results Directly.

Key Assumption Linear growth region dominates process. (most valid in adverse pressure gradients)

Hypothesis. Transition occurs when $\frac{A(x)}{A_0} = e^n$ $n \approx 9$ $e^9 \approx 8100$ for <u>some</u> frequency

$A = |v'|$, or wave amplitude, etc.
$= |u'|$

Spatial problem: must check "all" frequencies.



What is $A(x)$ for given ω_r?

$$v' = \hat{v}(y)\, e^{i(\alpha_r x - \omega_r t)} \cdot e^{-\alpha_i x}$$

$$A = |v'| = |\hat{v}|\, e^{-\alpha_i x}$$

$$\ln A = \ln|\hat{v}| - \alpha_i x$$

$$\frac{d}{dx}(\ln A) = -\alpha_i$$

$Re_{\theta_\theta} > Re_{\theta_s}(\lambda, \tau)$

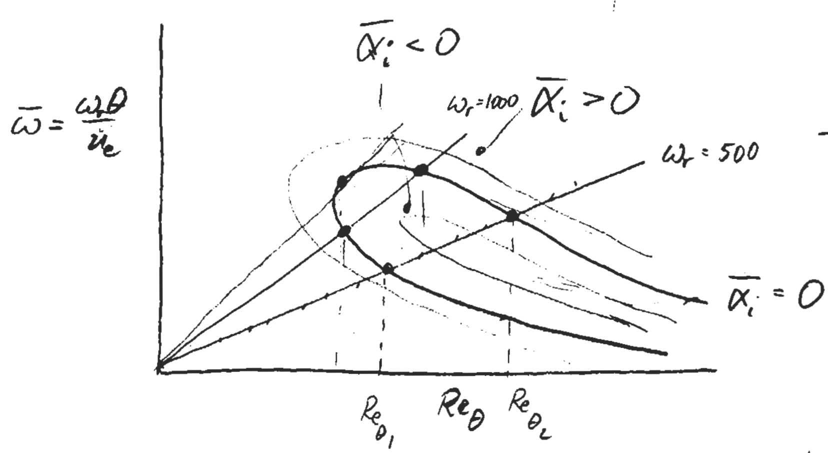
$\lambda = \frac{\theta^2}{\nu} \ldots$

$Re_{\theta_s} = 163 + \exp\, F(\lambda) \ldots$

$F = \frac{6.91 + 12.75\lambda + 13.\ldots\lambda^2}{6.91 + 7.48\lambda - 12.27\lambda^2}$

$$\alpha_i \cdot \theta = \bar{\alpha}_i(Re_\theta, H, \bar{\omega}) \qquad \bar{\omega} = \frac{\omega_r \theta}{u_e}$$

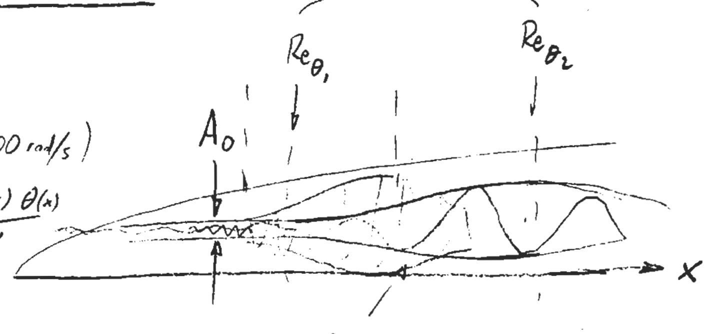
(Uses u_e, θ as ref velocity length)



Ex. Similar flow (H fixed)

$$\therefore \alpha_i = \frac{1}{\theta} \cdot \bar{\alpha}_i(Re_\theta, \bar{\omega})$$

$\alpha_i < 0$ (unstable)



Look at one frequency ω_r (say 500 rad/s)

know $\bar{\omega}(x) = \bar{\omega}_r \cdot \frac{\theta(x)}{u_e(x)}$, $Re_\theta(x) = \frac{u_e(x)\,\theta(x)}{\nu}$

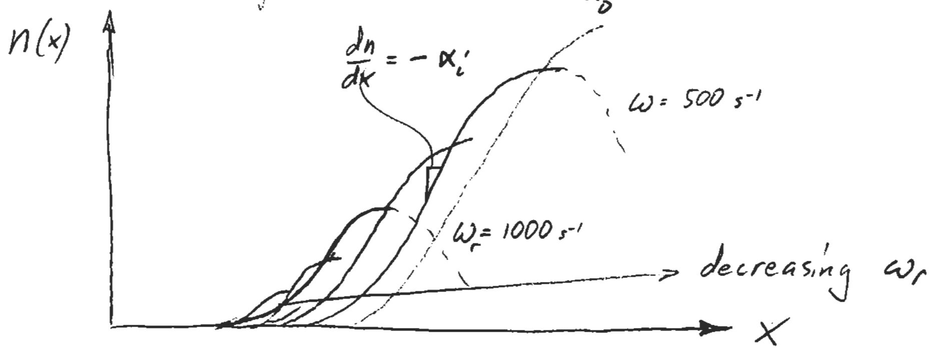
$$\frac{d}{dx}(\ln A) = -\alpha_i$$

<u>In general:</u>

$$\ln A(x;\omega_r) = \ln A_0(\omega_r) + \int_{x_0(\omega_r)}^{x} \frac{d}{d\xi}(\ln A)\, d\xi$$

location where $\alpha_i = 0$

$$\text{or} \quad \ln\left(\frac{A(x;\omega_r)}{A_0(\omega_r)}\right) \equiv n(x;\omega_r) = \int_{x_0}^{x} -\frac{1}{\theta(\xi)}\,\bar{\alpha}_i\left(Re_\theta(\xi),\, H(\xi),\, \omega_r \cdot \frac{\theta(\xi)}{u_e(\xi)}\right) d\xi$$



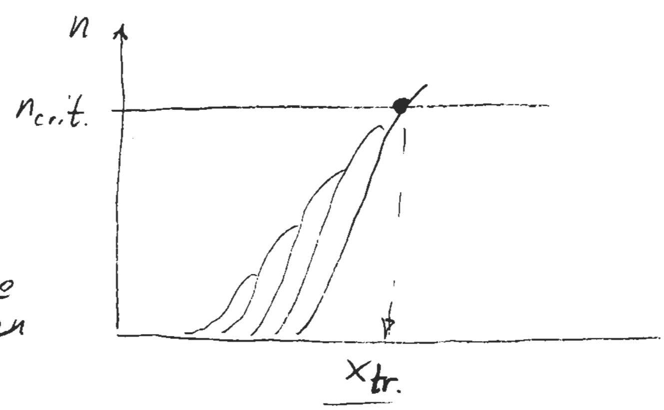
<u>In general</u> lower frequencies:

- go unstable farther downstream
- grow more (over a longer distance)

<u>Some</u> frequency will reach $n = n_{crit}$ first, triggering transition.

$$n_{crit} = \ln\left(\frac{A_{crit}}{A_0}\right) = \ln A_{crit} - \ln A_0$$

A_0 depends on ambient disturbance level. If A_{crit} is universal, then n_{crit} depends on dist. level.



	n_{crit}
sailplane	14 - 18
power plane	12
clean tunnel	8 - 10
dirty tunnel	4 - 8

e^n used to be called e^9 or e^{10} method.

← n_{crit} first determined in tunnel tests