Subsonic Small Disturbance Potential Flow

1. $\vec{V} = (V_\infty + \hat{u})\vec{i} + \hat{v}\vec{j}$ where $\underbrace{\frac{|\hat{u}^2 + \hat{v}^2|}{V_\infty^2} << 1}$

small disturbances are assumed

2. $\hat{u}\vec{i} + \hat{v}\vec{j} = \nabla\hat{\phi} \leftarrow \hat{\phi} \equiv$ perturbation potential

$$\Rightarrow \hat{u} = \frac{\partial\hat{\phi}}{\partial x} \qquad \hat{v} = \frac{\partial\hat{\phi}}{\partial y}$$

3. small-disturbance (?) and bc's=

$$(1 - M_\infty^2)\frac{\partial^2\hat{\phi}}{\partial x^2} + \frac{\partial^2\hat{\phi}}{\partial y^2} = 0$$

BC: $\hat{v}(x,0) = V_\infty \dfrac{dy_c}{dx}(x)$ where $y_c =$ camber line

Note: this eqn and bc are valid for subsonic and supersonic flow.

Also note, $C_p = \dfrac{p - p_u}{q_\infty} = -\dfrac{2\hat{u}}{V_\infty}$

4. For subsonic flow, we utilize the following mathematical transformation:
   Define:

$$\beta = \sqrt{1 - M_\infty^2} \qquad \begin{aligned} \xi &= x \\ \eta &= \beta y \\ \overline{\phi} &= \beta\hat{\phi} \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} &\frac{\partial^2\overline{\phi}}{\partial\xi^2} + \frac{\partial^2\overline{\phi}}{\partial\eta^2} = 0 \\ &\frac{\partial\overline{\phi}}{\partial\eta}(\xi,0) = V_\infty\frac{dy_c}{dx}(\xi) \end{aligned}$$

The implications are that the subsonic compressible flow around an airfoil can be related to the incompressible $(M_\infty = 0)$ flow about the airfoil.

	$M_\infty = 0$	$0 < M_\infty < 1$
Same eqn	$\dfrac{\partial^2\hat{\phi}}{\partial x^2} + \dfrac{\partial^2\hat{\phi}}{\partial y^2} = 0$	$\dfrac{\partial^2\overline{\phi}}{\partial\xi^2} + \dfrac{\partial^2\overline{\phi}}{\partial\eta^2} = 0$
Same bc	$\dfrac{\partial\hat{\phi}}{\partial y}(x,0) = V_\infty\dfrac{dy_c}{dx}(x)$	$\dfrac{\partial\overline{\phi}}{\partial\eta} = V_\infty\dfrac{dy_c}{dx}(\xi)$

Implications:

$$\hat{u}(x,y,M_\infty) = \underbrace{\frac{\partial\hat{\phi}}{\partial x}}_{\substack{M_\infty \neq 0 \\ \text{flow}}} = \frac{1}{\beta}\underbrace{\frac{\partial\overline{\phi}}{\partial \xi}}_{\substack{M_\infty = 0 \\ \text{flow}}} = \frac{1}{\beta}\overline{u}_0(\xi,\eta)$$

$$C_p(x,y,M_\infty) = -\frac{2\hat{u}}{V_\infty} = -\frac{1}{\beta}\underbrace{\frac{2\overline{u}_0}{V_\infty}}_{M_\infty = 0}$$

$$\Rightarrow \quad \boxed{C_p(x,y,M_\infty) = \frac{1}{\beta}C_{p_0}(\xi,\eta)}$$

What about the forces?