Method of Assumed Profiles

Here are the basic steps:

1. Assume some basic boundary velocity profile for $u(x,y)$. For example, this is a crude approach but illustrates the ideas:

$$\frac{u(x,y)}{u_e(x)} = \begin{cases} \dfrac{y}{\delta(x)}, 0 \le y < \delta(x) \\ 1, y \ge \delta(x) \end{cases}$$

where $\delta(x)$ is the single unknown describing the velocity distribution.

2. Calculate $\delta^*, \theta\,(or\,H)$, and C_f for the assumed profile:

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{u_e}\right) dy = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy = \left(y - \frac{1}{2}\frac{y^2}{\delta} \right)\Bigg|_0^\delta$$

$$\Rightarrow \boxed{\delta^* = \frac{1}{2}\delta}$$

$$\theta = \int_0^\infty \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right) dy = \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy = \left(\frac{1}{2}\frac{y^2}{\delta} - \frac{1}{3}\frac{y^3}{\delta^2} \right)_0^\delta$$

$$\Rightarrow \boxed{\theta = \frac{1}{6}\delta}$$

Note: $\boxed{H = \frac{\delta^*}{\theta} = \frac{\frac{1}{2}\delta}{\frac{1}{6}\delta} = 3}$

Finally, to find C_f we need $\tau_w = \mu \left.\dfrac{\partial u}{\partial y}\right|_{y=0}$

$$\frac{\partial u}{\partial y} = u_e \frac{\partial}{\partial y}\left(\frac{y}{\delta}\right) = \frac{u_e}{\delta}, \text{ for } 0 \le y < \delta$$

$$\Rightarrow \boxed{C_f = \frac{\tau_w}{\frac{1}{2}\rho_e u_e^2} = \frac{\mu \frac{u_e}{\delta}}{\frac{1}{2}\rho_e u_e^2} = \frac{2\mu}{\rho_e u_e \delta}}$$

3. Plug results from step 2 into integral b.l. equation:

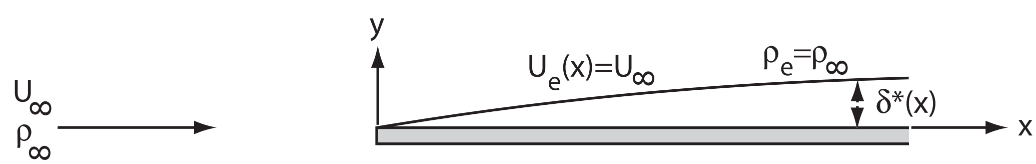
$$\frac{C_f}{2} = \frac{d\theta}{dx} + \frac{\theta}{u_e}(2+H)\frac{du_e}{dx}$$

So, for our assumed linear profile:

$$\boxed{\frac{\mu}{\rho_e u_e \delta} = \frac{1}{6}\frac{d\delta}{dx} + \frac{5\delta}{6u_e}\frac{du_e}{dx}} \qquad (1)$$

where $\delta(x)$ is the only unknown. We can solve this by specifying $u_e(x)$, setting an initial value for δ at $x = 0$ (i.e. the leading edge) and then integrate in x. Note: in many cases, this integration will need to be done numerically.

<u>Example: Flat Plate</u>



Since $u_e(x) = u_\infty$ is a constant, the governing equation (1) becomes:

$$\frac{\mu}{\rho_\infty u_\infty \delta} = \frac{1}{6}\frac{d\delta}{dx}$$

Re-arrange and integrate:

$$\frac{6\mu}{\rho_\infty U_\infty} = \delta \frac{d\delta}{dx}$$

$$\frac{6\mu}{\rho_\infty U_\infty} = \frac{1}{2}\frac{d(\delta^2)}{dx}$$

$$\int_0^x \frac{6\mu}{\rho_\infty U_\infty} dx = \frac{1}{2}\int_0^x \frac{d(\delta^2)}{dx} dx$$

$$\frac{6\mu}{\rho_\infty U_\infty} x = \frac{1}{2}\left[\delta^2(x) - \delta^2(0)\right]$$

But, our initial condition is $\delta(0) = 0$.

$$\frac{6\mu}{\rho_\infty U_\infty} x = \frac{1}{2}\delta^2(x)$$

$$\Rightarrow \boxed{\frac{\delta}{x} = \sqrt{\frac{12\mu}{\rho_\infty U_\infty x}} = \frac{2\sqrt{3}}{\sqrt{\text{Re}_x}} = \frac{3.464}{\sqrt{\text{Re}_x}}}$$

$$\Rightarrow \frac{\delta^*}{x} = \frac{\frac{1}{2}\delta}{x} = \frac{1}{2}\frac{2\sqrt{3}}{\sqrt{\text{Re}_x}}$$

$$\boxed{\frac{\delta^*}{x} = \frac{\sqrt{3}}{\sqrt{\text{Re}_x}} = \frac{1.732}{\sqrt{\text{Re}_x}}}$$

and C_f:

$$C_f = \frac{2\mu}{\rho_e u_e \delta} = \frac{2\mu}{\rho_\infty u_\infty x}\frac{\sqrt{\text{Re}_x}}{2\sqrt{3}}$$

$$C_f = \frac{1}{\sqrt{3}\sqrt{\text{Re}_x}}$$

$$\boxed{C_f = \frac{0.577}{\sqrt{\text{Re}_x}}}$$

Comparison with Blasius solution:

	Blasius	Int. Method with linear velocity
$\frac{\delta^*}{x}\sqrt{\mathrm{Re}_x}$	1.720	1.732
$C_f\sqrt{\mathrm{Re}_x}$	0.664	0.577