NAME : ....

Massachusetts Institute of Technology

16.07 Dynamics

Problem Set 10

Out date: Nov. 7, 2007

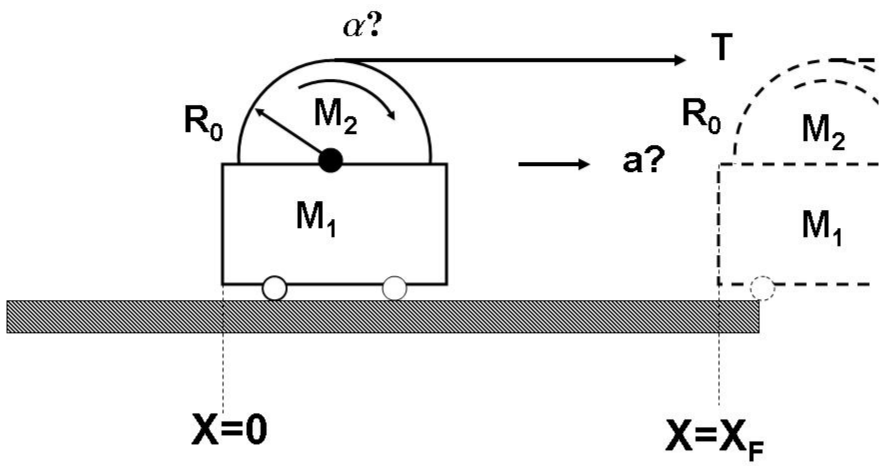
Due date: Nov. 14, 2007

	Time Spent [minutes]
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Study Time	

Turn in **each problem** *on* **separate sheets** *so that grading can be done in parallel*

Problem 1 (10 points)

Question 1



A cart of mass M_1 rolls on bearings on a frictionless plane. It carries a large disk of radius R_0 mounted by an axle to the cart at its center of mass. The disk is uniform and of mass M_2. It is free to rotate without friction about its axle. A rope is wrapped around the disk as sketched. Everything is at rest. At $t = 0$ a constant tension T is applied to the rope.

a) What is the linear acceleration of the cart, a?

b) What is the angular acceleration of the disk α?

c) The rope with constant tension T pulls the cart for a distance of X_F meters. How much work is done by the external force T?

d) Assuming a conservative system (no friction etc.) what is the kinetic energy when the cart is located at X_F?

e) Does this all make sense?

(Note: the radius of gyration of a uniform circular disk is $R_0/\sqrt{2}$)

Problem 2 (10 Points)

A uniform bar of length L and mass m is lying on a frictionless horizontal surface. It is pinned at a pivot-point O a distance h from its end. An impulse P_0 is applied to its end. There is a reaction impulse P_R exerted on the bar by the pivot.

a) What is the velocity of the center of mass v_G in terms of the impulses P_O and P_R.

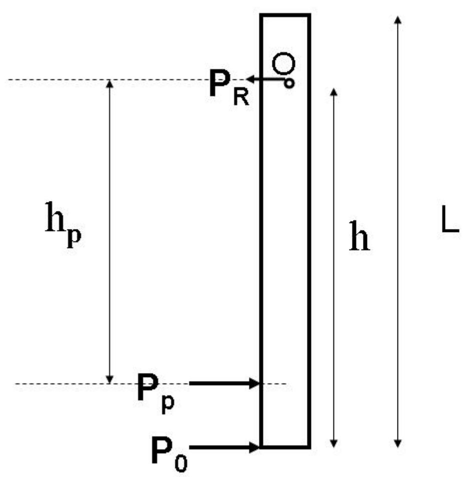
b)What is the angular velocity $\dot{\theta}$ in terms of v_G?

c) What is the moment of inertia of the bar about the pivot point O, I_O?

d) What is the angular velocity of the bar, $\dot{\theta}$, in term of the impulses P_0 and P_R

e) How would you solve for the reaction impulse P_R in the pivot?

f) Now consider a slightly different problem. Is there a point h_P on the bar (see sketch) at which an impulse P_P could act without creating a reaction impulse P_R?
(Note: the radius of gyration of a uniform bar about its center of mass is $k_G = L/\sqrt{12}$. The parallel axis theorem states that the moment of inertia about any point a distance r_0 from the center of mass G is $I_{r_0} = I_G + mr_0^2$.)



Problem 3(10 points)

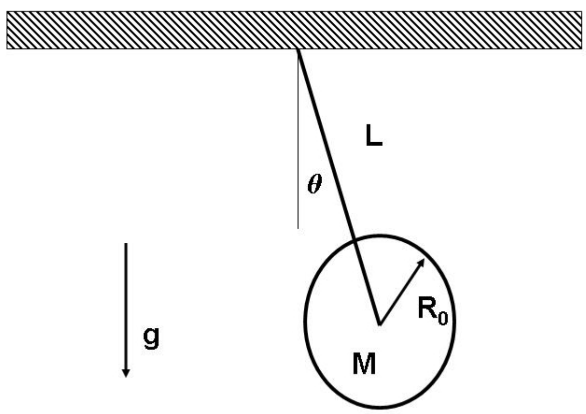
A uniform circular disk of radius R_0 and mass M is suspended from a pivot by a massless rod of length L between the pivot and the center of the disk to which it is rigidly fixed/attached. The disk is displaced through an angle θ_0 as shown and then released at $t = 0$. Consider small angles in your discussion.
Note: the radius of gyration of a uniform circular disk is $R_0\ \sqrt{2}$.

a) What is the resulting motion for $t > 0.$?

b) What is the governing differential equation for $\ddot{\theta}$?
c) What are the boundary conditions to be applied to this differential equation?
d) What is the solution to the differential equation?
Now consider the problem using energy. Use second-order small-angle approximations to express your results to $O(\theta^2)$.
e) What is the maximum potential energy for an angular displacement θ_0?

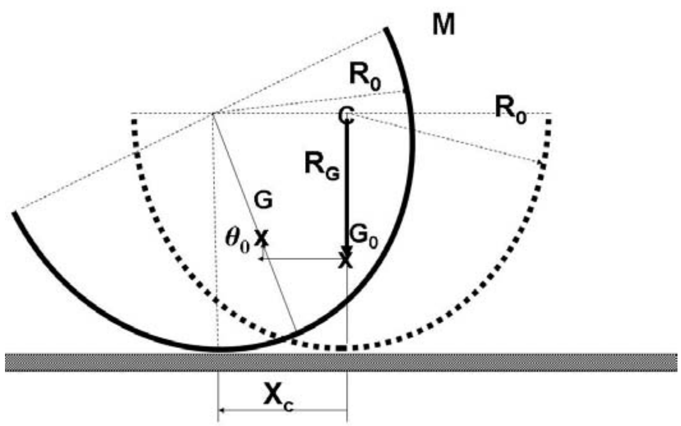
f) What is the maximum kinetic energy? Where in the motion does that occur?

g) What is the frequency of oscillation?
h) Does this agree with the frequency found in part d)?



Problem 4(10 points)

A thin 2D semi-circular arc of mass M and radius R_0 rests on a flat plane. It is displaced through an angle θ_0 and held at rest. It maintains rolling contact with the plane. (i.e. it rolls without slipping.). The location of the center of mass for the arc is $R_G = 2R_0/\pi$.



a) What is the moment of inertia of the semicircular arc about its center, I_C?
b) Using the parallel axis theorem, find the moment of inertia about the center of mass, I_G.
c) What is the displacement of the point of contact, X_C?
As it rolls, the center of mass displaces in both the horizontal and vertical directions.
d)What is the vertical displacement of the center of mass, y_{CM}?
e) What is the horizontal displacement of the center of mass x_{CM}?
f) What is the potential energy in the displaced $\theta = \theta_0$ position with the arc at rest?
Now the arc is released and returns to and past $\theta = 0$ in an oscillatory motion.
g) What is the total kinetic energy in both translation and rotation as the arc passes through $\theta = 0$. Remember that the center of mass will take up a position exactly on the alternate side at the completion of that phase of the motion, so it needs to move. This has implications for total kinetic energy.
h) Equating the maximum potential energy to the maximum kinetic energy, what is the natural frequency of oscillation of the arc?

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