16.06 Principles of Automatic Control
Lecture 15

The Negative (0°) Root Locus

Sometimes, we need to plot the root locus for a negative gain parameter

Example:

Longitudinal dynamics of 747, M=0.8 at 20,000 ft.
h =altitude
δe =elevator deflection

$$\frac{h(s)}{\delta e(s)} = \frac{32.7(s+0.0045)(s+5.645)(s-5.61)}{s(s+0.003\pm 0.0098j)(s+0.6463\pm 1.1211j)}$$

Poles of system:

- $s=0$ "energy mode" - represents change in total (kinetic plus potential) energy. Hard to control with elevator - hence near cancellation with zero at $s=-0.0045$.
- $s=-0.6463\pm 1.1211j$ "short period mode." The short period mode is dominated by changes in aircraft pitch altitude, much like an arrow or weather vane feathering into the wind.
- $s=-0.003\pm 0.0098j$ "phugoid mode". This mode represents long period exchange of kinetic and potential energy, with very small changes in aircraft angle of attack.

Note that:

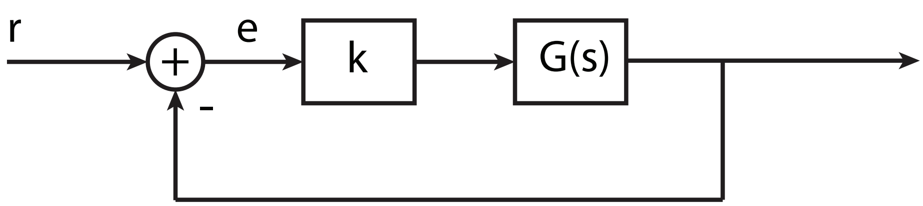
1. There is a two orders of magnitude difference between the natural frequencies of the short period and phugoid modes.
2. The phugoid has only the modest damping ($\zeta=0.29$). Often, it is much lower, say, only a few percent.

For our discussion, the most interesting aspect is the right half plane zero at $s = +5.61$. Why?

Rewrite the transfer function as

$$\frac{h(s)}{\delta e(s)} = G(s) = \underset{\text{root locus gain of G(s)}}{\underset{\downarrow}{32.7}} L(s)$$

Look at the feedback loop using only proportional gain[1]:



So the characteristic equation is:

$$0 = 1 + kG(s) = 1 + \underbrace{32.7k}_{K} L(s)$$

The question is, should k, (and therefore K) be positive or negative?

Note that because there are an off number of RHP poles and zeros, the D.C. gain $G(0)$ is negative. To improve performance at D.C., must have *negative* gain.

There are other situations where a negative R.L. gain may be required, but this is the most common.

Modification to the Root Locus Rules.

The fundamental result is that s is on the negative locus if

$$1 + KL(s) = 0 \Rightarrow L(s) = -\frac{1}{K}$$

for some negative K. If K is negative, $-1/K$ is positive, so that the angle condition is

[1]For this system, a PD controller would be better, but that is not important for our argument.

$$\angle L(s) = 0^\circ + l \cdot 360^\circ, \quad l \text{integer}$$

So the rules are:

- **Rule 1**: The n branches of the locus start at the n poles. m approach the zeros, n-m approach ∞. (No change)
- **Rule 2**: The locus is on the real axis to the left of an even number of poles and zeros.
- **Rule 3**: The asymptotes are described by

$$\begin{aligned} \alpha =& \frac{\sum p_i - \sum z_i}{n-m}, \quad \text{(no change)} \\ \theta_l =& \frac{360^\circ \cdot (l-1)}{n-m}, \quad l = 1, 2, 3, ...n-m \end{aligned}$$

Note: no 180° term.

- **Rule 4**: The departure angles from poles and the arrival angles at poles are given by

$$\begin{aligned} \phi_{\text{dep}} =& \frac{\sum \Psi_i - \sum^* \phi_i - 360^\circ \cdot (l-1)}{q} \\ \Psi_{\text{arr}} =& \frac{\sum \phi_i - \sum^* \Psi_i + 360^\circ \cdot (l-1)}{q} \end{aligned}$$

where q is multiplicity of pole or zero, $l = 1, 2, ...q$.

- **Rule 5:** The locus crosses the imaginary axis for values of K at which Routh's criterion shows a change in the number of unstable poles. (No change)
- **Rule 6:** No change in rule for when there are multiple points on the locus.

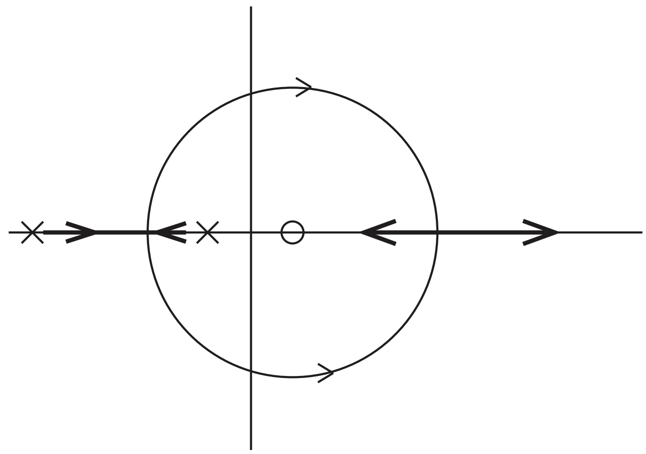
In summary, all rules are the same, except:

1. All $180^\circ s$ become $0^\circ s$.
2. "Odd" becomes "even" in Rule 1.

Example

$$G(s) = \frac{s-1}{(s+1)(s+3)}$$

0° locus:



Note:

- Locus looks familiar but *is* different
- RHP zero tends to pull poles into RHP - bad.

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