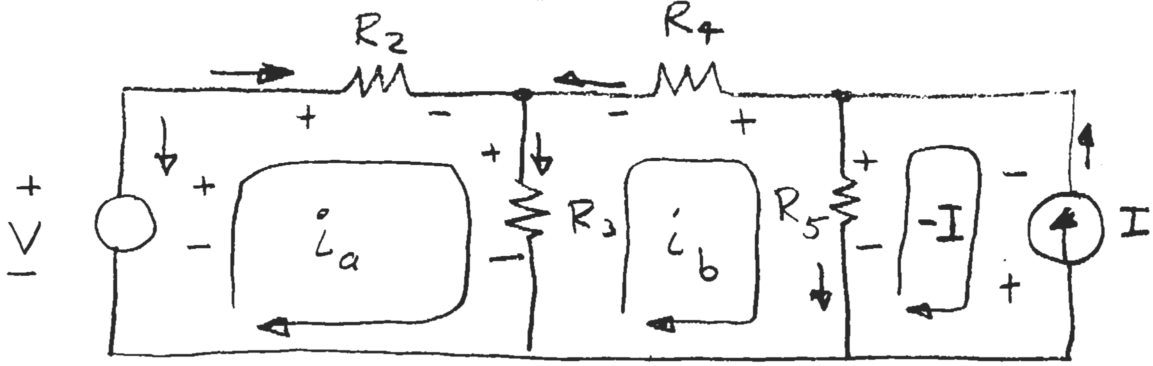
Lecture 55

The Loop Method

The loop method is an alternative method for solving networks. The basic idea is to choose current variables that automatically satisfy KCL at each node



Steps:

1. Identify as many independent loops as possible.

$$\# \text{loops} = \# \text{elements} - \# \text{nodes} + 1$$
$$= 6 - 4 + 1 = \underline{\underline{3}}$$

2. Label the loop currents, using variables or known current values

$i_a,\ i_b,\ -I$

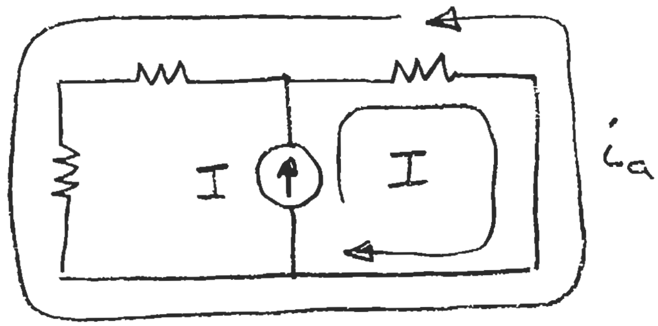
Be careful here! Notation means that

$i_1 = -i_a$ $\qquad$ $i_4 = -i_b$

$i_2 = i_a$ $\qquad$ $i_5 = i_b + I$

$i_3 = i_a - i_b$ $\qquad$ $i_6 = I$

So we want only one loop to touch current source. E.g.,



3. KCL is automatically satisfied.

4. Apply KVL around each loop, looking for voltage drop around loop.

$$i_a: \; -V + i_2 R_2 - i_3 R_3 = 0$$

$$= -V + i_a R_2 - (i_a - i_b) R_3$$

$$= (R_2 + R_3) i_a - R_3 i_b - V = 0$$

$$\Rightarrow \underbrace{(R_2 + R_3)}_{\Sigma R\text{'s around loop}} i_a \; - R_3 i_b = V$$

V ← voltage source in loop.

R_3 ↑ R bordering i_a, i_b

[Do loop method concept test here]

Around loop i_b,

$$i_b: \; -R_3 i_a + (R_3 + R_4 + R_5)\, i_b = -R_5 I$$

5. Collect equations and solve:

$$\begin{bmatrix} R_2 + R_3 & -R_3 \\ -R_3 & R_3 + R_4 + R_5 \end{bmatrix} \begin{Bmatrix} i_a \\ i_b \end{Bmatrix} = \begin{Bmatrix} V \\ -R_5 I \end{Bmatrix}$$

Use Cramer's rule:

$$i_a = \frac{\begin{vmatrix} V & -R_3 \\ -R_5 I & R_3 + R_4 + R_5 \end{vmatrix}}{\begin{vmatrix} R_2 + R_3 & -R_3 \\ -R_3 & R_3 + R_4 + R_5 \end{vmatrix}}$$

$$= \frac{(R_3 + R_4 + R_5) V - R_3 R_5 I}{(R_2 + R_3)(R_3 + R_4 + R_5) - R_3^2} = a_1 V + a_2 I$$

$$i_b = \frac{R_3 V - (R_2 + R_3) R_5 I}{(R_2 + R_3)(R_3 + R_4 + R_5) - R_3^2} = \underbrace{b_1 V + b_2 I}$$

Note: Solution is <u>linear</u> in V, I. This means <u>superposition</u> holds.

[Do loop method, node method CTs here]