

## Muddy Card Responses Lecture M13 2/22/2004

Synopsis. Reviewed work on buckling at start of lecture. Re-introduced concept of structural integrity. Will look at material aspects of this, particularly those associated with failure and longevity. Performed demonstrations of plastic and yield behavior of metals and also introduced the possibility of fracture (sudden catastrophic failure). Drew prototypical stress-strain curve for a ductile metal. Identified key features: yield stress, tensile strength, elastic unloading after yield, work hardening, permanent plastic strain after yield. Identified the need for use of "true" stress and strain when dealing with large deformations since cross-sectional areas and reference lengths change with deformation. Identified grouping of strength-related behaviors by material class, also observed wide range of values of strength taken by materials of a particular type (eg. Aluminum alloys can have yield strengths from 50MPa-700 MPa). Will explore why this is next time.

**Distinction between true stress and stress somewhat unclear.** Not sure where to start with this. Please read the notes and give me a starting point.

**Why is it that there is a point where you continue to increase force and area in compression but stress and strain become constant?** I realize that I made a misleading statement at this point of the lecture. The curve on the compression side does not stop because of the stress-strain response. Other factors come in to play. In particular friction between the compression plates and the specimen places a limit on the load that can be applied.

**Muddy: For  $\epsilon_t$  vs.  $\sigma_t$  there is a lower limit to  $\epsilon_t$  but if you keep compressing is this limit eventually broken? Like ice if you keep compressing it changes states...Is that just a breakdown for this model?** Sort of, see above -friction is the most usual limit here. pressure-induced phase changes are well outside the scope of what we will discuss in Unified – but this is a real phenomenon. Ice is a particularly complicated material.

**How did you calculate work? Is it  $W = \int F \cdot D$ ?**  $\sigma = \frac{F}{A}$ ,  $\epsilon = \frac{\Delta L}{L}$ ,  $V = AL$ , hence

$W = \int \sigma d\epsilon \cdot V$ . This is exactly correct. I could not have put it better. The one thing to add is that internally some of this work goes into elastic strain energy, which is recovered as the load is taken off the specimen, the rest goes into heat creation.

**To what extent can Poisson's ratio be used to model necking.** Poisson's ratio (and the Young's modulus) only applies for elastic behavior. In plasticity we will use the knowledge that volume is conserved.

**How do dynamics play a role? It seems qualitatively that many materials fail faster when loaded more quickly.** Good question. There are several mechanisms that may have some rate-dependent component. We will just touch on creep (time and temperature dependent deformation), which is one source of rate dependence in metals. We have already seen that polymers are visco-elastic material, and so we would expect to see less viscous behavior at higher rates. In metals such as aluminum, titanium or steel the rate dependences are not significant at room temperature, unless one goes to very high strain rates, when inertia effects at the atomic level start to play a role.

**I understood polymers to undergo changes in their crystal structure as they necked. With the necked region become less amorphous. Is this true?** Sort of. Polymers don't really have a crystal structure (although they may have regions which are somewhat ordered that are called "crystalline"). As they are plastically deformed in tension the polymer chains will align and this will tend to increase the tensile strength. As a result necking in polymers can be stable whereas in metals it is generally not.

**How do so called "memory metals" not deform plastically nearly as soon as regular metals? Guess: Lots of pre-straining during manufacture?** These are usually called "shape memory alloys" Nickel-Titanium (Nitanol) is the most common. These materials have two stable crystal structures, which have different packing densities. Swapping between one crystal structure and the other results in a volume change and therefore a strain results. Switching between crystal structures (phases) can occur by application of stress or temperature. The phase change (switching) is essentially elastic (some times it is called hyperelastic) and can result in large strains compared to the conventional elastic limit of metals 2-6% strain are possible, depending on the alloy composition, its microstructure etc. The materials, although elastic, are quite non-linear in their stress strain response, so this limits their applications.

**Why must volume be conserved in plastic deformation? Obviously mass is conserved, so this implies the density cannot change. Is this because if the density changed the material properties would change thus it would no longer be plastic deformation?** At this stage in our discussion of plastic deformation this is just an experimental result. In lecture M16 we will examine the mechanisms of plastic deformation and understand this a bit better.

**Question: Hard-panning is due increasing  $\sigma_y$  of a material?** I do not understand this question. I am guessing it is asking about work hardening. This is the phenomenon that the yield stress of a material is observed to increase with the amount of plastic strain that the material experiences.

**Mud: Could you please derive the relations:  $\sigma_t = \sigma_n (1 + \epsilon_n)$**

$\epsilon_t = \ln(1 + \epsilon_n)$ . I think that you are being a little lazy here, but here goes

Load/actual cross-sectional area:  $\sigma_t = F/A$  (vs  $\sigma_{nominal} = F/A_0$ )

**True strain,  $\epsilon_t$  defined incrementally:**

$$\delta \varepsilon = \frac{\delta u}{l} = \frac{\delta l}{l} \Rightarrow d\varepsilon = \frac{dl}{l}$$

Hence strain developed from an initial length  $l$  to a final length  $l_0$  is given by:

$$\varepsilon_t = \int_{l_0}^l \frac{dl}{l} = \ln\left(\frac{l}{l_0}\right)$$

Also, in a plastic deformation volume is conserved (material is incompressible). Hence, if the

elastic deformation is negligible,  $A = \frac{A_0 l_0}{l}$ . Hence,

$$\sigma_t = \frac{F}{A} = \frac{Fl}{A_0 l_0}$$

Rearranging and substituting: **HERE IS THE MISSING LINE!!!**

$$\varepsilon_n = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1 \Rightarrow \frac{l}{l_0} = 1 + \varepsilon_n$$

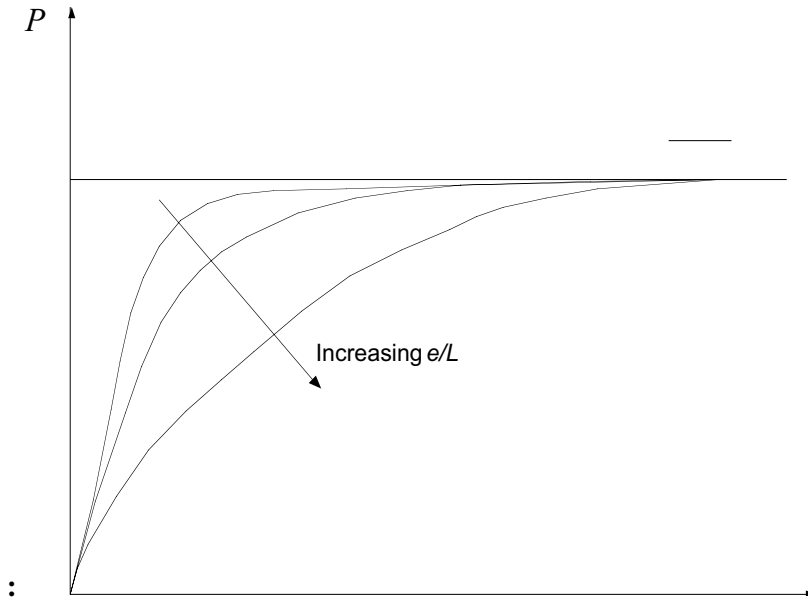
Hence

$$\sigma_t = \sigma_n (1 + \varepsilon_n)$$

And

$$\varepsilon_t = \ln(1 + \varepsilon_n)$$

**With the graph**



I'm not sure what the two paths that it can take are. It can either go straight up or to one side? YES this is correct. But in reality it will never go straight up due to imperfections, so then it could go either right or left (you never draw the graph to the left but can it go that way too?) so there are really four paths: up-right/left or side – right/left. OK, I understand. Your initial statement is correct. The two equilibrium paths are (1) it remains straight above  $P_{crit}$  and continues to deform axially or (2) it buckles off to one side or the other (whether it goes left or right doesn't matter, it is the same equilibrium path. Secondly, if there is an imperfection then it is determined ahead of time whether it will deform to the right or the left – there is no uncertainty. I only draw the right hand side because of this – simply because of the way that I defined the initial imperfection to result in a positive  $w(x)$ . If I defined a "negative" initial imperfection then I would have drawn the curves to the left of the axis.

**Also, it probably is always the same group of people turning in the muddy cares. That doesn't mean no one else has mud or is totally lost either, but usually we realize what we don't know while working on the problem set. That 's why professors at office hours is so valuable.** Thank you very much for the feedback., and also for evidence that someone is reading this muddy responses!. If nothing else I find them useful as a way of keeping track of reactions to lectures and the concepts within them.

**Dinner conversations must be quite exciting at your house.** Interesting comment. I am intrigued to know what is behind this.

There were 2 muddy cards with no mud, or positive responses.