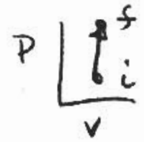
SOLUTIONS TO UNIFIED T2 (WAITZ)

a) GIVEN TO PROPERTIES FOR INITIAL COND. THEREFORE STATE IS DEFINED.

$$v_i = \frac{RT_i}{P_i} \qquad R_{He} = \frac{\mathcal{R}}{MW} = \frac{8.314}{4} = 2.078 \frac{kJ}{kg\text{-}K}$$

$$v_i = \frac{(2.078\times10^3)(300)}{100\times10^6} = 0.0062 \frac{m^3}{kg}$$

GIVEN PATH TO FINAL STATE (RIGID TANK $\Rightarrow \Delta v = 0$)



AND GIVEN FINAL TEMPERATURE.

SO $v_f = v_i = 0.0062$, $T_f = 400K$

$$\therefore \; P_f = \frac{RT_f}{v_f} = \frac{(2078)400}{0.0062} = 133 MPa$$

b) NO WORK WAS DONE. $\int p\,dv = 0$ SINCE $dv = 0$

c) HERE A NEW PATH IS SPECIFIED, INSTEAD OF V = CONST, WE HAVE $\frac{dp}{dv} = 1\times10^5 \frac{MPa}{m^3/kg}$ $\therefore$ $p = 1\times10^5 v + \text{Const.}$

SOLVE FOR CONSTANT USING P_i, v_i, YOU GET

$$p = 1\times10^5 \frac{MPa}{m^3/kg} \cdot v - 523.4 \text{ MPa}$$ } NEW PATH FULLY DEFINED

BUT NOT GIVEN P_f OR v_f, INSTEAD WE ARE GIVEN T_f SO SUBSTITUTE USING IDEAL GAS

$$p = \frac{RT_f}{v_f} \quad \text{(UNITS Pa)}$$

$$\frac{2078 \cdot 400}{v_f} = 1\times10^{11} v_f - 523.4\times10^6$$

(NOTE EXTRA 10^6 TO MAKE UNITS CONSISTENT)

REWRITING, $V_f^2 - 5.234\times10^{-3}\,V_f - 8.314\times10^{-6} = 0$

BY QUADRATIC FORMULA $V_f = \dfrac{5.234\times10^{-3} \pm 7.788\times10^{-3}}{2}$

$$\boxed{V_f = 0.00651\ \frac{m^3}{kg}}$$

$$P_f = \frac{RT_f}{V_f} = \frac{2078.400}{0.00651} = \boxed{127.7\,MPa}$$

d) $W = \int_{V_1}^{V_2} p\,dv = \int_{V_1}^{V_2} \left(1\times10^5 \frac{MPa}{m^3/kg}\cdot V - 523.4\,MPa\right) dV$

$$W = \frac{1\times10^{11}}{2}V^2 - 523.4\times10^6\,V \Big|_{V_1}^{V_2}$$

$$\boxed{W = 31.4\ kJ/kg}$$

e)