a) $C_D = c_d + \underbrace{\left(\frac{C_L^2}{\pi AR}\right)}_{\equiv C_{D_i}}$ , $\frac{C_L}{C_D} = \frac{C_L}{c_d + C_L^2/\pi AR}$

Maximize $\frac{C_L}{C_D}$ $\rightarrow$ set $\frac{d}{dC_L}\left(\frac{C_L}{C_D}\right) = 0$

$$\frac{d}{dC_L}\left(\frac{C_L}{c_d + C_L^2/\pi AR}\right) = \frac{(c_d + C_L^2/\pi AR) - C_L(2C_L/\pi AR)}{(c_d + C_L^2/\pi AR)^2} = \frac{c_d - C_L^2/\pi AR}{(\quad)^2} = 0$$

$\Rightarrow$ $c_d = \frac{C_L^2}{\pi AR}$ or $\boxed{C_L = \sqrt{c_d \pi AR}}$ at max $\frac{C_L}{C_D}$

At this point, $\boxed{C_D = c_d + \frac{(\sqrt{c_d \pi AR})^2}{\pi AR} = 2c_d}$ $C_{D_i} = c_d$

b) Set $\frac{d}{dC_L}\left(\frac{C_L^{3/2}}{C_D}\right) = 0$

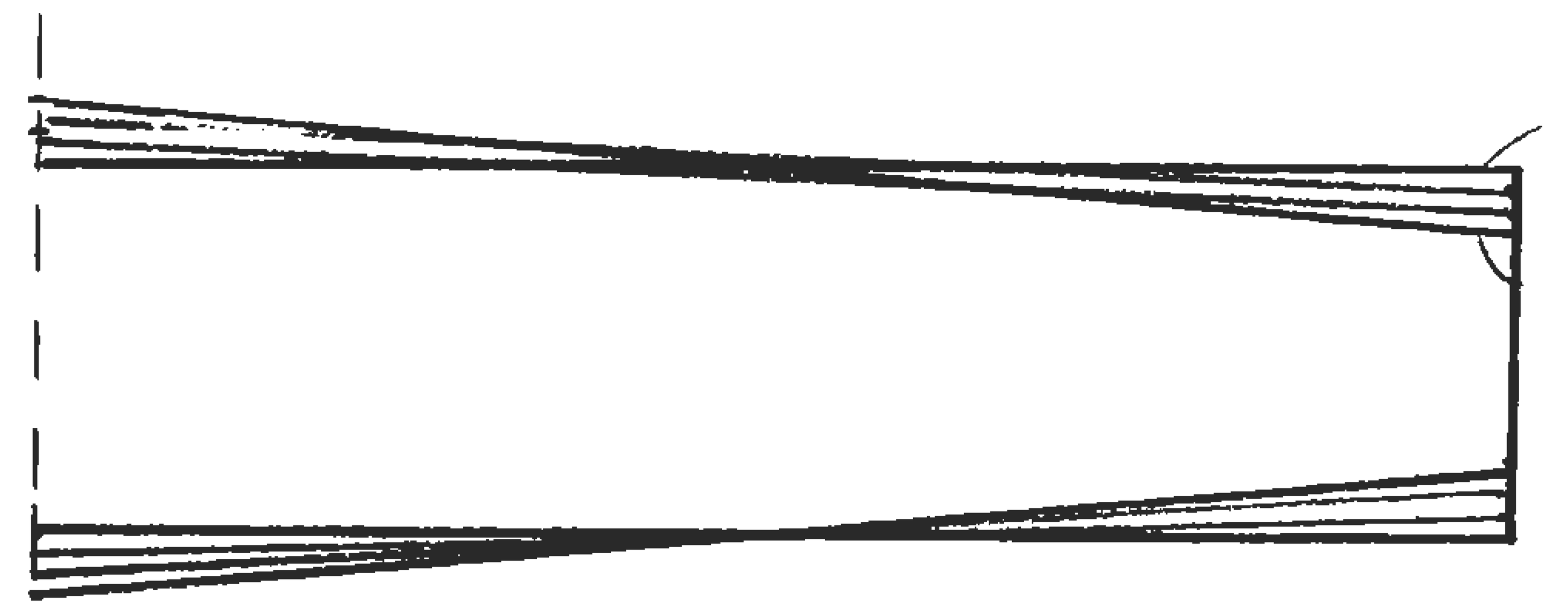
$$\frac{d}{dC_L}\left(\frac{C_L^{3/2}}{c_d + C_L^2/\pi AR}\right) = \frac{\frac{3}{2}C_L^{1/2}(c_d + C_L^2/\pi AR) - C_L^{3/2}(2C_L/\pi AR)}{(c_d + C_L^2/\pi AR)^2} = C_L^{1/2}\,\frac{\frac{3}{2}c_d - \frac{1}{2}\frac{C_L^2}{\pi AR}}{(\quad)^2} = 0$$

$\Rightarrow$ $3c_d = \frac{C_L^2}{\pi AR}$ or $\boxed{C_L = \sqrt{3 c_d \pi AR}}$ at max $\frac{C_L}{C_D}$

At this point, $\boxed{C_D = c_d + \frac{(\sqrt{3 c_d \pi AR})^2}{\pi AR} = 4c_d}$ $C_{d_i} = 3c_d$

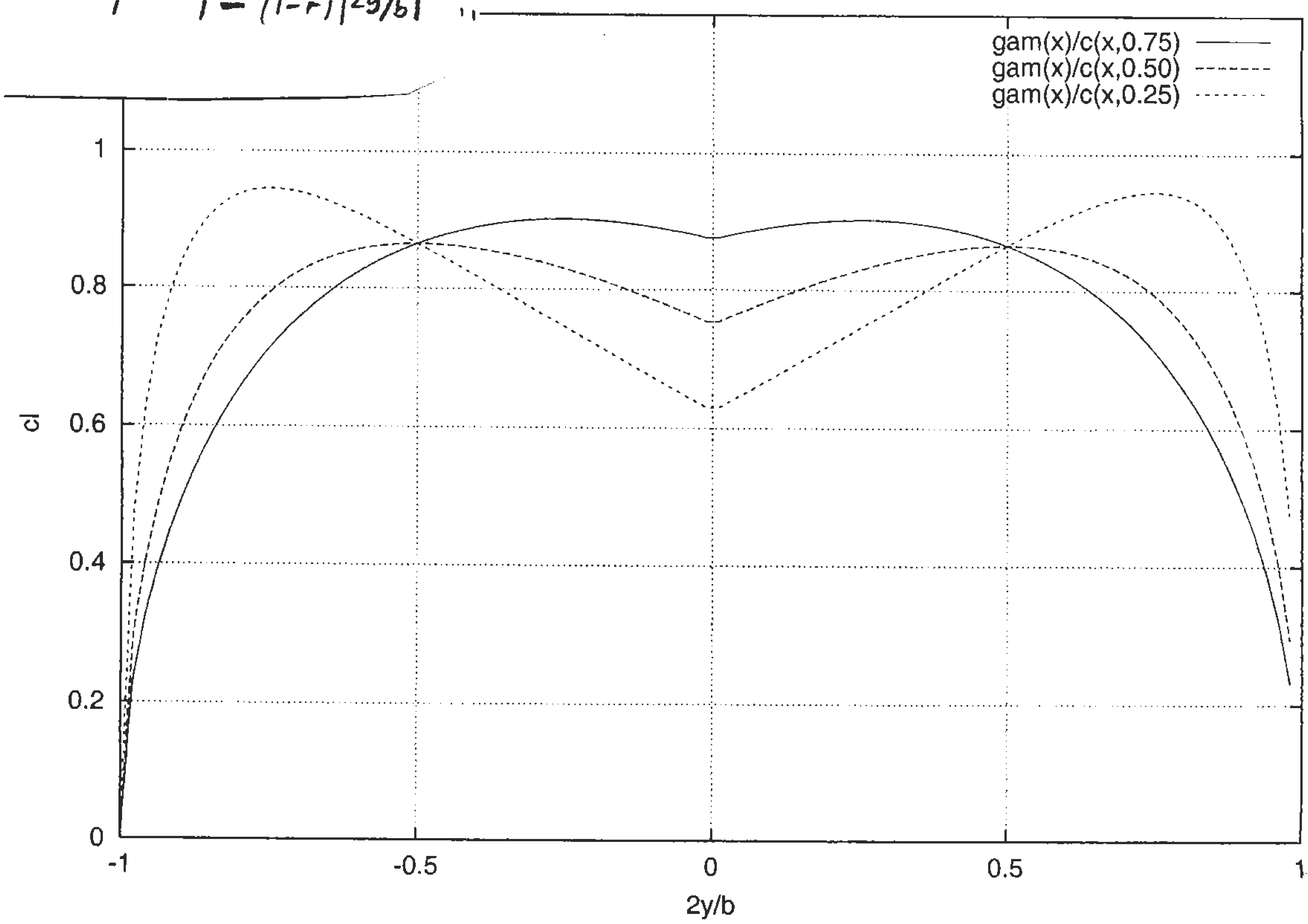
a) Linear taper from $c_r$ to $c_t$ : $c(y) = c_r + (c_t - c_r)\left|\frac{2y}{b}\right|$

or $c(y) = \frac{2}{1+r} c_{avg}\left[1 - (1-r)\left|\frac{2y}{b}\right|\right]$

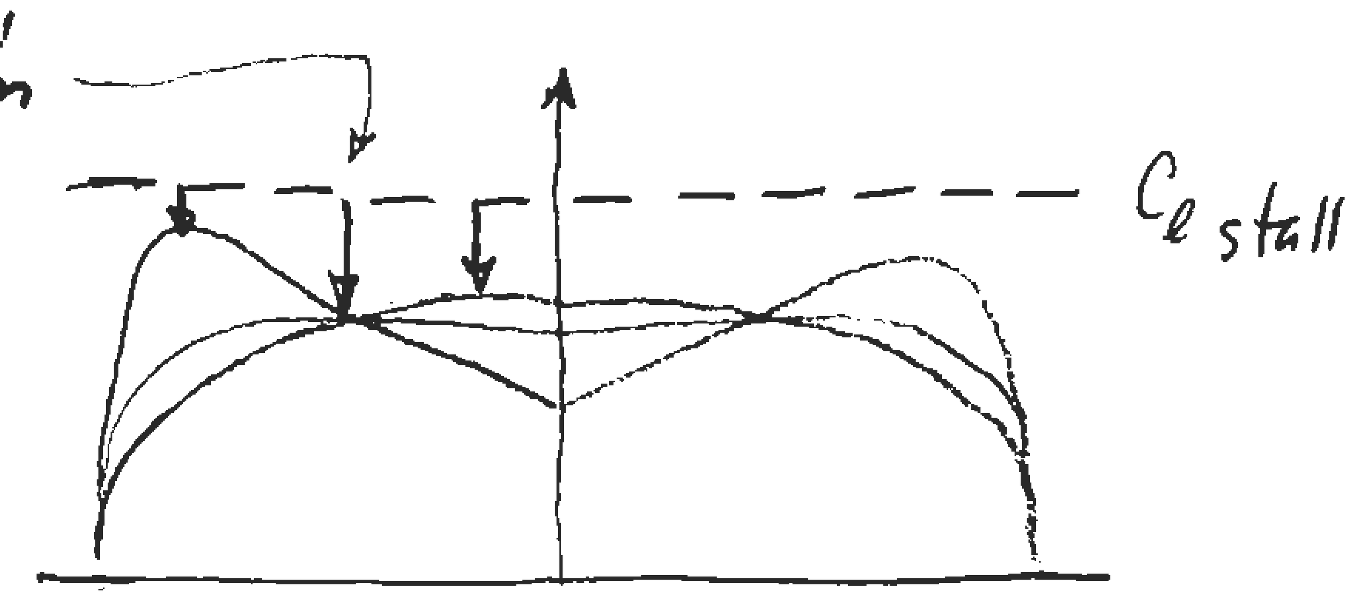



b) $\Gamma(y) = \frac{1}{2} V c(y)\, c_\ell(y) \Rightarrow c_\ell(y) = \frac{2\Gamma(y)}{V_\infty c(y)} = \frac{2\Gamma_0}{V_\infty c_{avg}} \frac{1+r}{2} \frac{\sqrt{1-(2y/b)^2}}{1-(1-r)|2y/b|}$

Plots of $\frac{\sqrt{1-(2y/b)^2}}{1-(1-r)|2y/b|}$



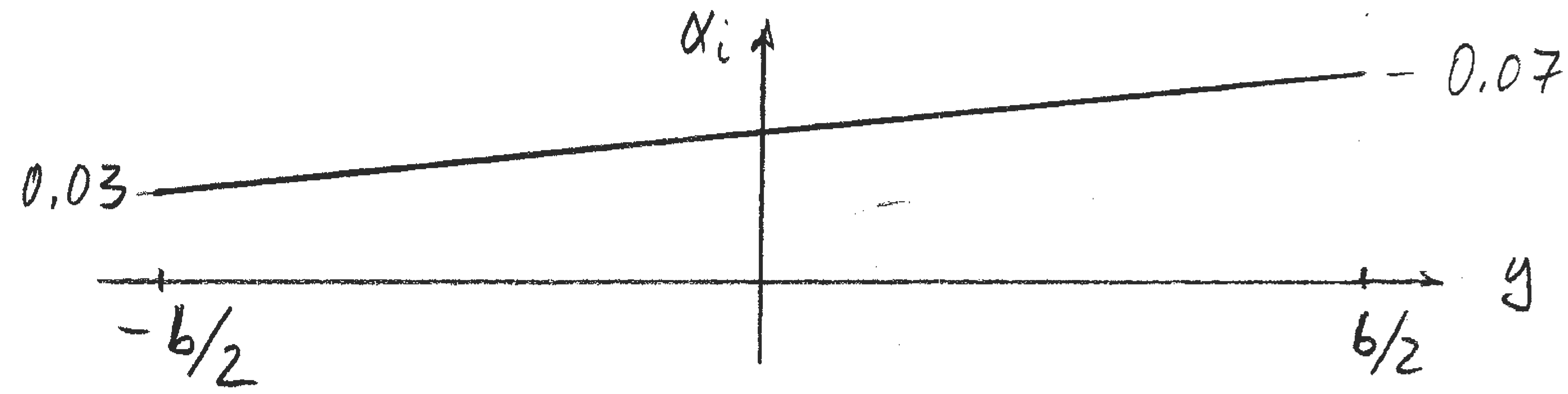

The middle $r = 0.50$ case has the smallest $(c_\ell)_{max}/C_L$, so it has the largest stall margin



UE Fluids | F9 + F10 Solution | Spring '04

a) $\alpha_i = \sum n A_n \frac{\sin n\theta}{\sin\theta} = A_1 + A_2 \frac{\sin 2\theta}{\sin\theta} = A_1 + A_2\, 2\cos\theta$

but since $\cos\theta = \frac{2y}{b}$, $\alpha_i = 0.05 + 0.02 \cdot \left(\frac{2y}{b}\right)$




b) $M_{roll} = \int_{-b/2}^{b/2} \rho V_\infty \Gamma y \, dy$

Using $\Gamma = 2b_\infty V_\infty (A_1 \sin\theta + A_2 \sin 2\theta)$

$y = \frac{b}{2}\cos\theta$

$dy = -\frac{b}{2}\sin\theta \, d\theta$

$$M_{roll} = \int_{\pi}^{0} \rho V_\infty \, 2bV_\infty (A_1 \sin\theta + A_2 \sin 2\theta) \frac{b}{2}\cos\theta \left(-\frac{b}{2}\sin\theta \, d\theta\right)$$

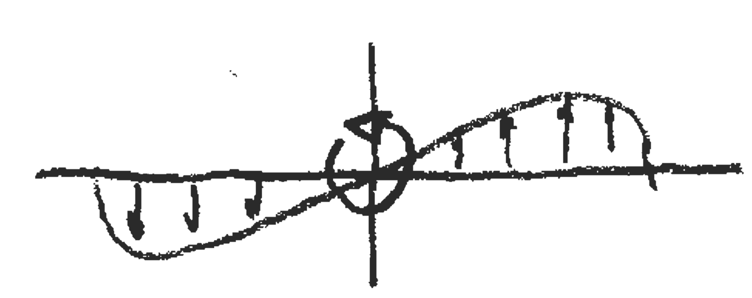
$$= \frac{1}{2}\rho V_\infty^2 b^3 \int_0^{\pi} (A_1 \sin\theta + A_2 \sin 2\theta) \frac{1}{2}\sin 2\theta \, d\theta$$

since $\cos\theta \sin\theta = \frac{1}{2}\sin 2\theta$

$$= \frac{1}{2}\rho V_\infty^2 b^3 \left[\frac{1}{2}A_1 \underbrace{\int_0^{\pi} \sin\theta \sin 2\theta \, d\theta}_{0} + \frac{1}{2}A_2 \underbrace{\int_0^{\pi} \sin 2\theta \sin 2\theta \, d\theta}_{\frac{\pi}{2}}\right]$$

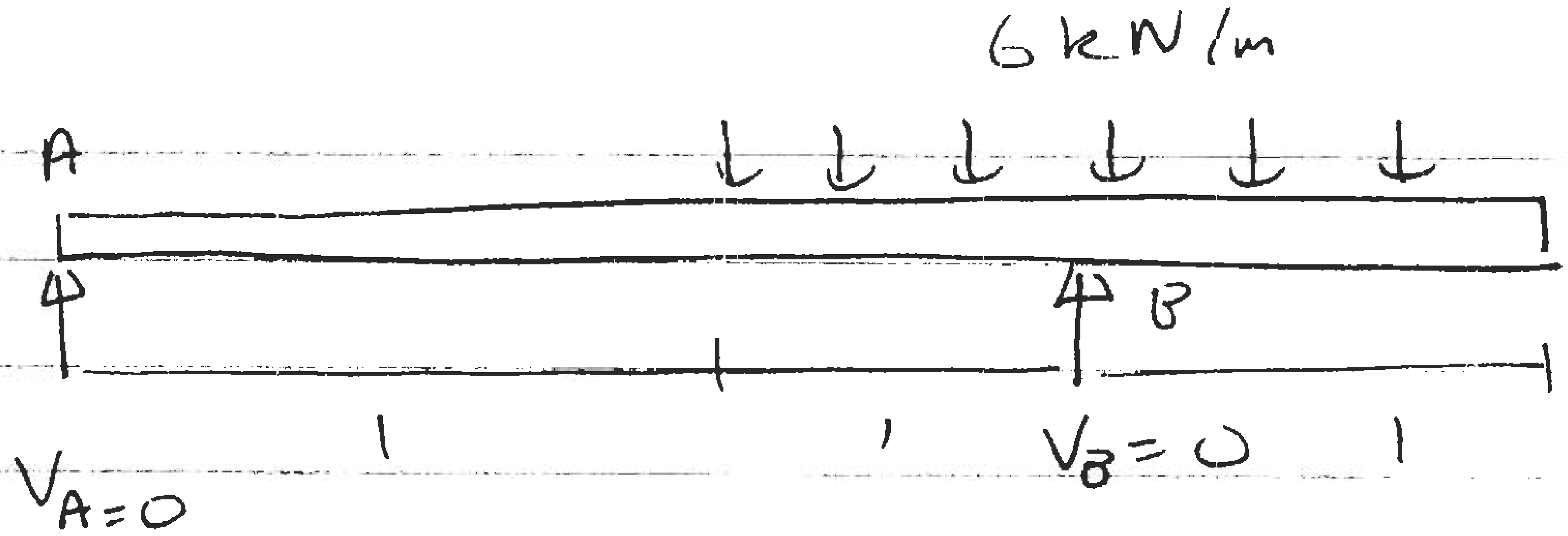
$$M_{roll} = \frac{\pi}{8}\rho V_\infty^2 b^3 \cdot A_2$$

only $A_2$ term contributes to rolling moment

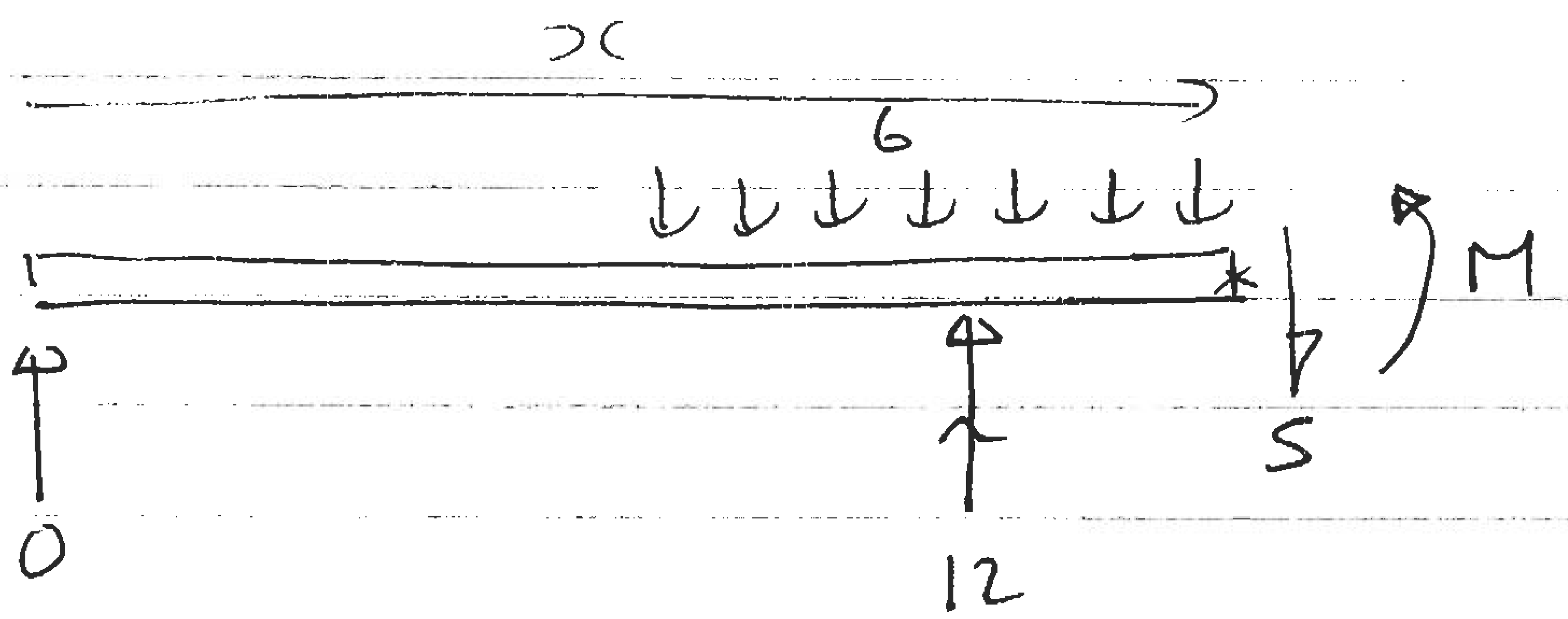


M7/M8

a)




Apply Macauley's method




$\circlearrowleft \Sigma M_x = 0$: $M + 6\frac{\{x-1\}^2}{2} - 12\{x-2\} = 0$

$M = -6\frac{\{x-1\}^2}{2} + 12\{x-2\}$

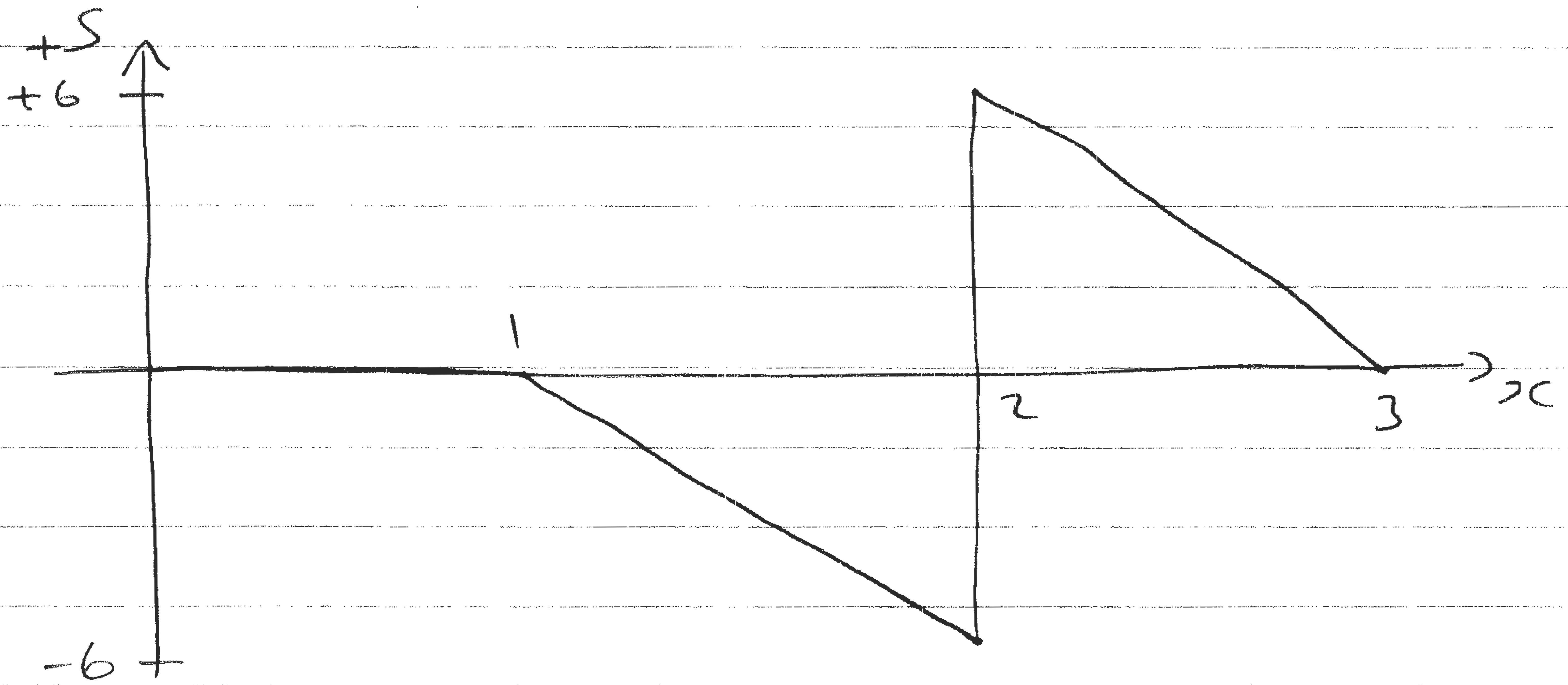
$\cancel{\Sigma F_z \uparrow = 0}$:

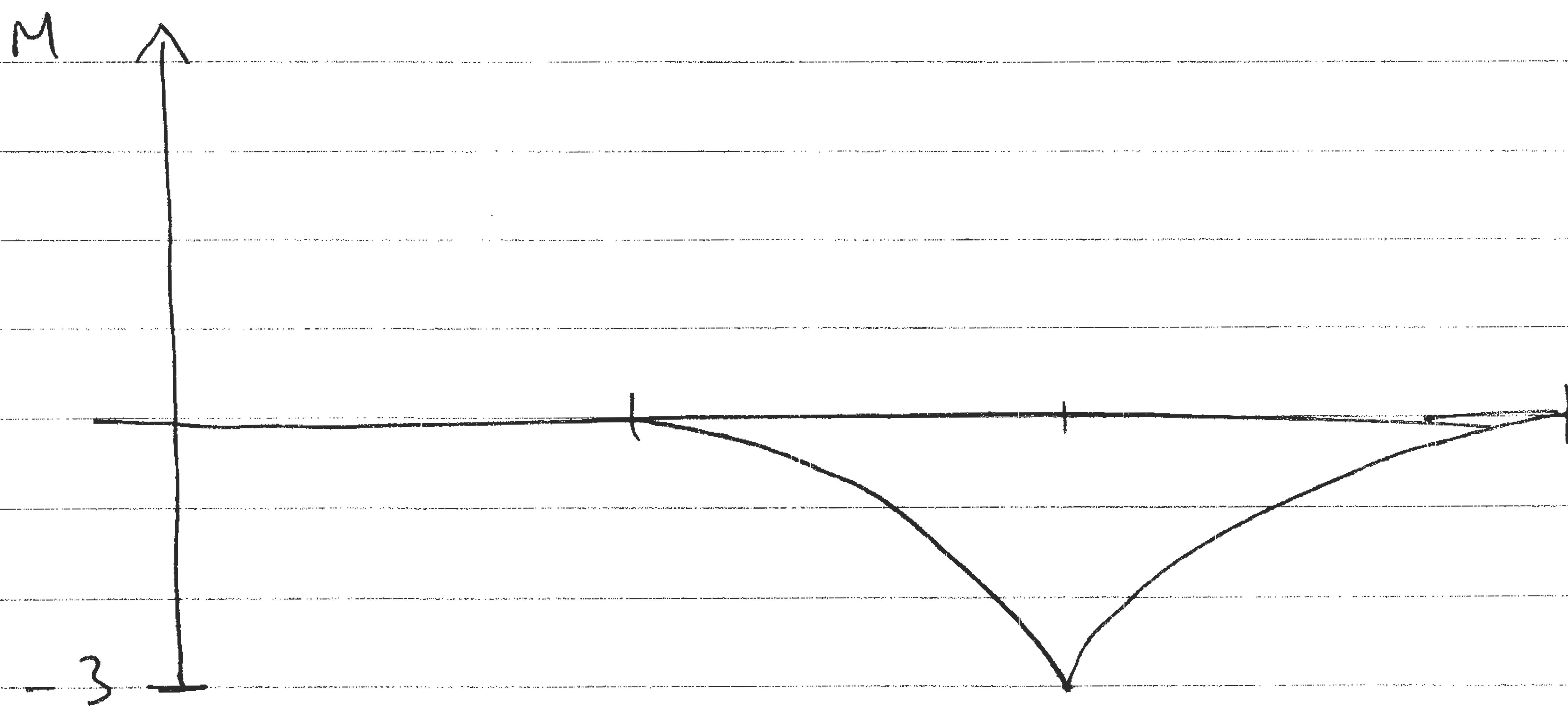
$\cancel{-S - 6\{x-1\} + 12 = 0}$

$\cancel{S = -6\{x-1\} + 12}$

from $\cancel{M = -EI\frac{d^2y}{dx^2}}$ $S = \frac{dM}{dx}$

$$S = \frac{dM}{dx} = +6\{x-1\} + 12\{x-2\}^0$$



6)

$$EI\frac{d^2w}{dx^2} = -3\{x-1\}^2 + 12\{x-2\}$$

$$EI\frac{dw}{dx} = \frac{-3\{x-1\}^3}{3} + \frac{12\{x-2\}^2}{2} + A$$

$$EIw = -\frac{\{x-1\}^4}{4} + \frac{\cancel{12}6\{x-2\}^3}{3} + Ax + B$$

apply boundary conditions

$w = 0$ @ $x = 0$, $x = 2$.

@ $x = 0$ $\Rightarrow$ $B = 0$ $\{x-1\}, \{x-2\} = 0$

@ $x = 2$: $-\frac{1}{4} + A \cancel{x} 2 = 0$

$$\therefore A = \frac{1}{8}$$

$$EIw = -\frac{\{x-1\}^4}{4} + 2\{x-2\}^3 + \frac{x}{8} \Leftarrow$$

Maximum deflection occurs either

for $0 < x < 2$ or at $x = \cancel{4}\ 3$

for $0 < x < 1$ $\quad EI\frac{dw}{dx} = \cancel{A}\frac{1}{8}$ $\therefore$ no min/max

for $1 < x < 2$

$$EI\frac{dw}{dx} = -\cancel{3}\{x-1\}^3 + \cancel{A}\frac{1}{8}$$

$$-x^3 + 3x^2 - 3x + \frac{9}{8} \cancel{+\frac{1}{8}} = 0$$

Solution $x = 1.04$.

$$\Rightarrow EIw = \frac{-(0.04)^4}{4} + \frac{1.04}{8} = +0.13\,\cancel{EI} \Leftarrow$$

$$w = \frac{0.13}{EI} \Leftarrow \quad @\ x = 1.04\,m.$$

This is the maximum upbending

$$w = \frac{0.13\times10^3}{3.54\times10^6} = 36.7\,\mu m \Leftarrow$$

kNm

for $x = 3$ ~~4~~

$$EIw = -\frac{\{2^4)}{4} + 2\{1\}^3 + \frac{3}{8}$$

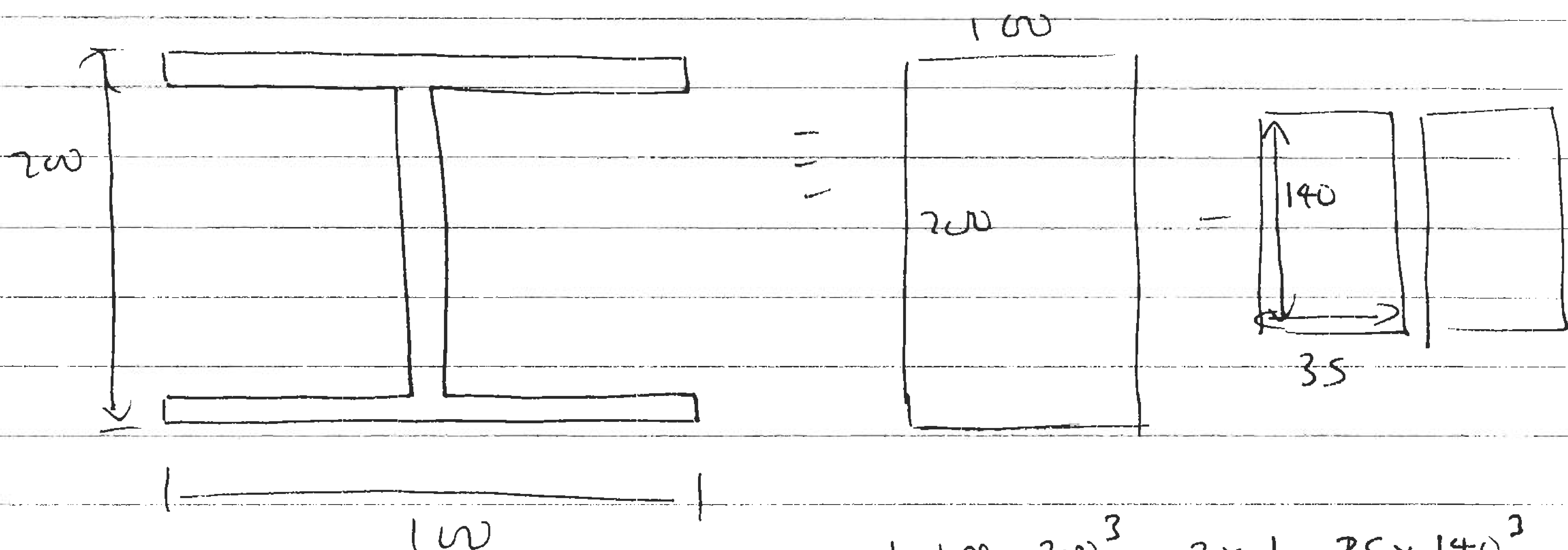
$$EIw = -4 + 2 + \frac{3}{8} \qquad w = \frac{-13}{8EI}$$

$$= \frac{-1.625}{EI}$$

This is the maximum down bending @ $x = 3$ m.

$w = -459$ μm. (0.459 mm.)

Calculation of I




$$= \frac{1}{12} 100 \times 200^3 - 2 \times \frac{1}{12} \times 35 \times 140^3$$

$$= 50.7 \times 10^6 \text{ mm}^4$$

$$EI = 70 \times 10^9 \times 50.7 \times 10^6 \times 10^{-12} = 3.54 \times 10^6 \text{ Nm}^2$$

$= -459$

c)

$\sigma_{xx} = \frac{-Mz}{I}$ maximum at max bending moment, max $z$ @ $x = 2$, $M = 3$ kNm

@ $\frac{h}{2} = \pm 100$ mm

$\therefore \sigma_{xx} = \frac{3\times10^{3} \times 100 \times 10^{-3}}{50\cdot7\times10^{6}\times10^{-12}} = 5.9$ MPa

Now no danger of yield due to tensile stresses

$\sigma_{xz} = \frac{-SQ}{Ib}$. $Q = \int_{z}^{h/2} z \, dA$

will be maximum at center of beam. ie. for $z = 0$

$$Q = \int_{0}^{70} z \, . \, 30 \, dz + \int_{70}^{100} z \, . \, 100 \, dz$$

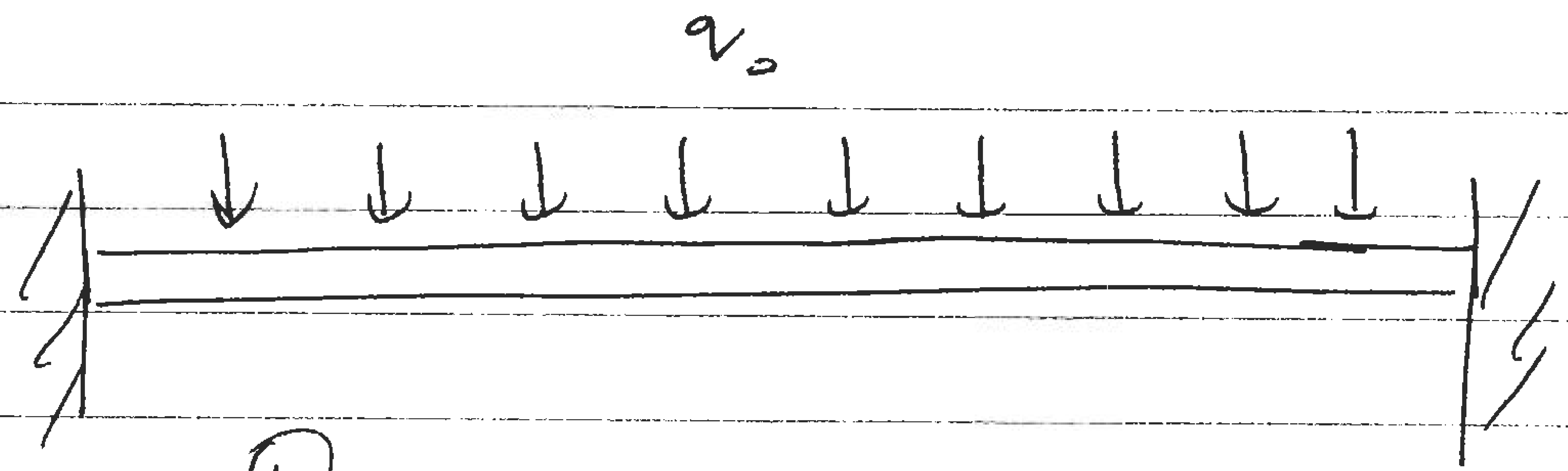
$$= \left[\frac{30z^2}{2}\right]_0^{70} + \left[\frac{\cancel{100}z^2}{2}\right]_{70}^{100} = 328\cdot5\times10^{3} \text{ mm}^3$$

$$\therefore \sigma_{xz} = -\frac{6\times10^{3} \times 328\cdot5\times10^{3} \times \cancel{10^{-9}}}{50\cdot7\times\cancel{10^{6}}\cancel{\times10^{-12}}\times 30\times\cancel{10^{-3}}} = 1.29\times10^{6} \text{ MPa}$$

Note I beam shear stress $\approx$ bending stress

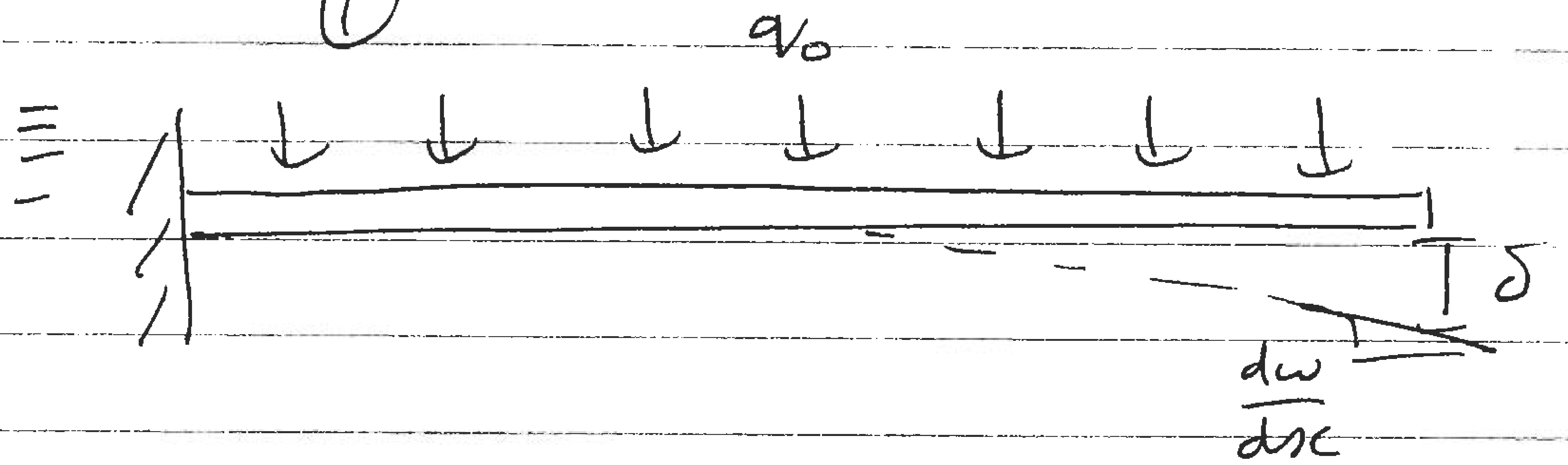
Yield is not a problem.

M9



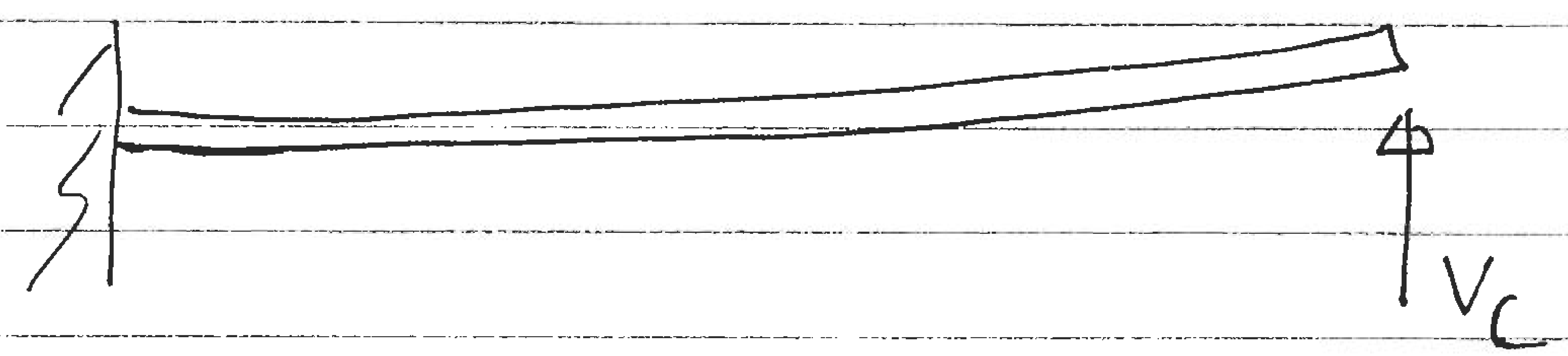

Statically indet.

(1)



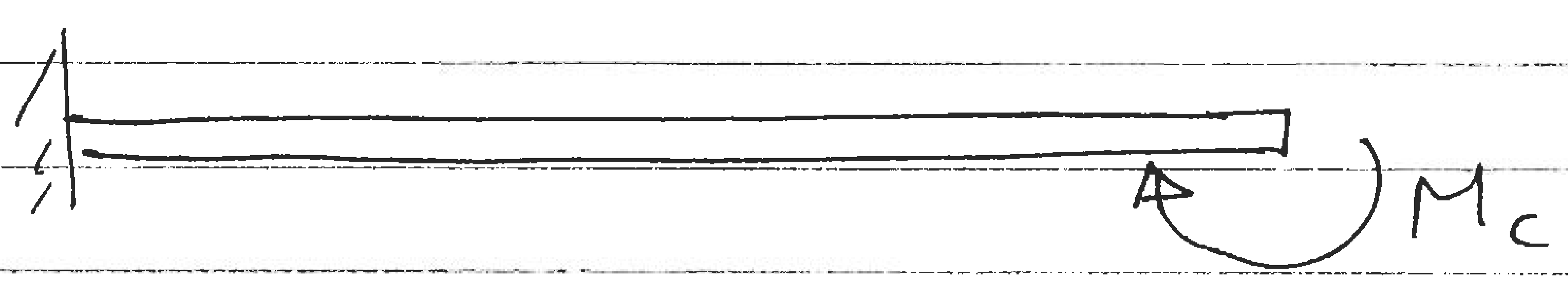

$$\delta^1 = \frac{q_0 L^4}{8EI} \qquad \frac{dw^1}{dx} = \frac{q_0 L^3}{6EI}$$

(2)




$$\delta^2 = -\frac{V_c L^3}{3EI} \qquad \frac{dw^2}{dx} = \frac{-V_c L^2}{2EI}$$

(3)




$$\delta^3 = \frac{M_c L^2}{2EI} \qquad \frac{dw^3}{dx} = \frac{M_c L}{EI}$$

Match B.C.'s at RHE, $x = L$  $w = 0$  $\frac{dw}{dx} = 0$

$w = 0$ : $\delta^1 + \delta^2 + \delta^3 = 0$

$$\frac{L^2}{EI}\left(\frac{q_0 L^2}{8} - \frac{V_c L}{3} + \frac{M_c}{2}\right) = 0$$

$$12 M_c - 8 V_c L + 3 q_0 L^2 = 0 \quad (1)$$

$\frac{dw}{dx} = 0$  $\frac{dw^1}{dx} + \frac{dw^2}{dx} + \frac{dw^3}{dx} = 0$

$$\frac{L}{EI}\left(\frac{q_0 L^2}{6} - \frac{V_c L}{2} + M_c\right) = 0$$

$$6 M_c - 3 V_c L + q_0 L^2 = 0 \quad (2)$$

Multiply (2) by 2 and subtract from (1)

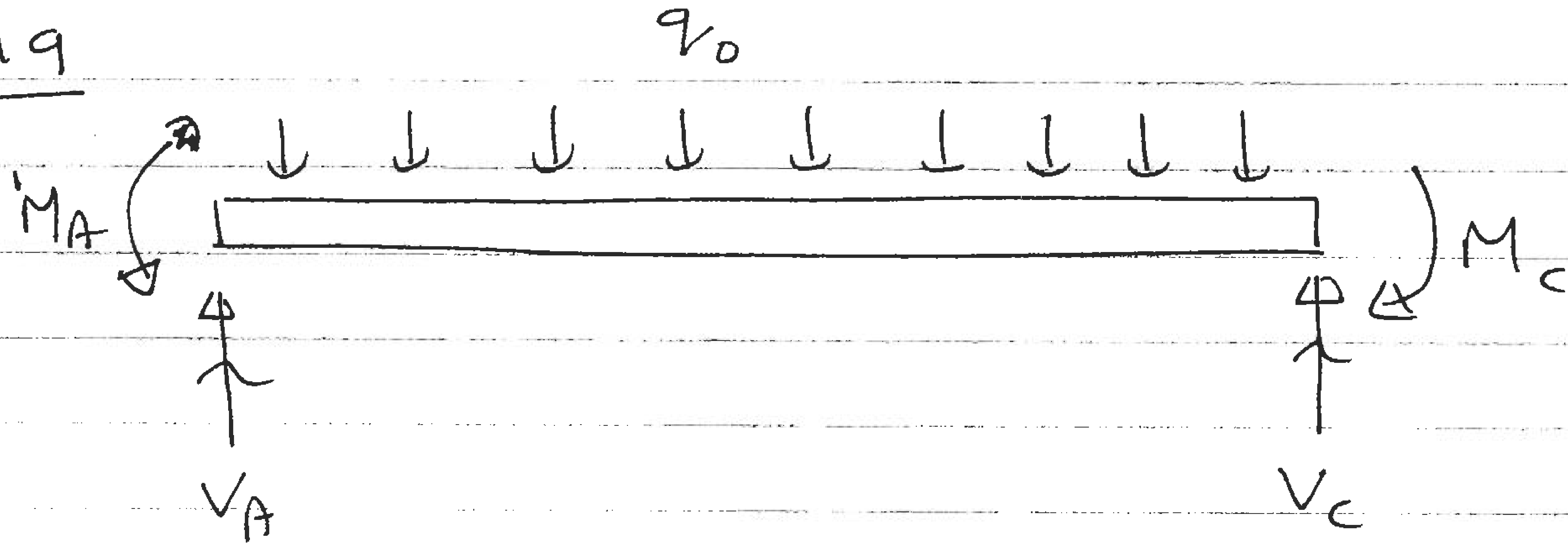
$$0 - 2 V_c L + q_0 L^2 = 0$$

$$V_A = V_c = \frac{q_0 L}{2} = \frac{q_0 L}{2} \Leftarrow \quad (!!)$$

Substitute back into (2)

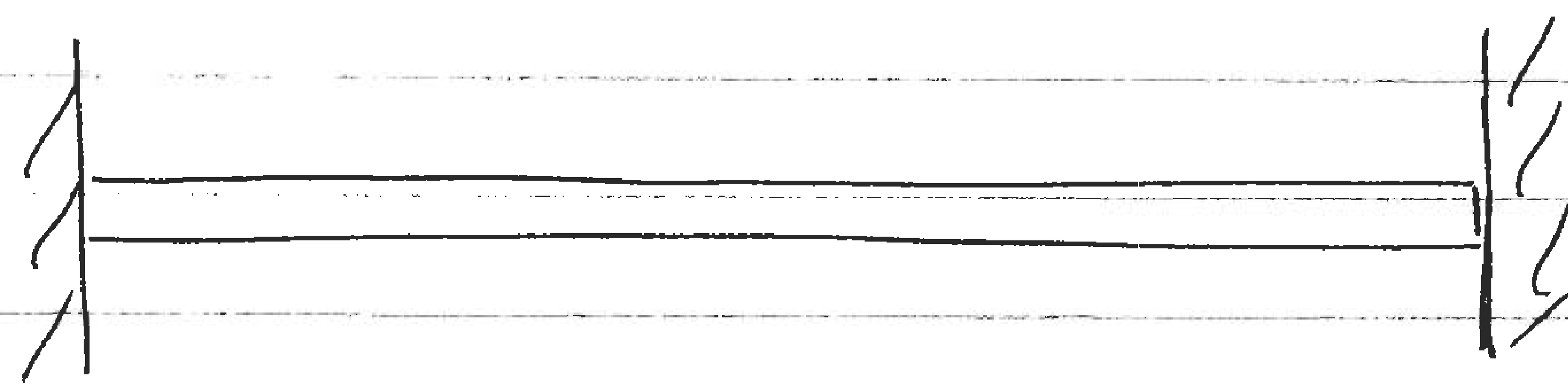
$$6 M_c - \frac{3 q_0 L^2}{2} + q_0 L^2 = 0 \; : \; M_c = M_A = \frac{q_0 L^2}{12} \Leftarrow$$

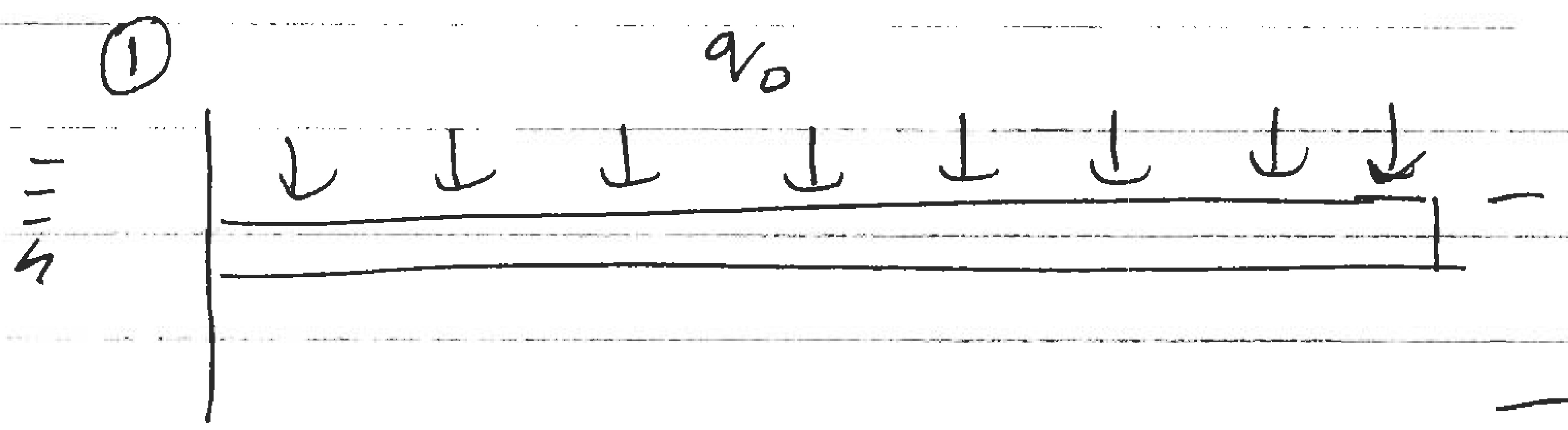
M9




By symmetry $M_A = M_c$
$V_A = V_c$.

But






$$\delta = \frac{q_0 L^4}{8EI} \quad \frac{dw}{d}$$

dw