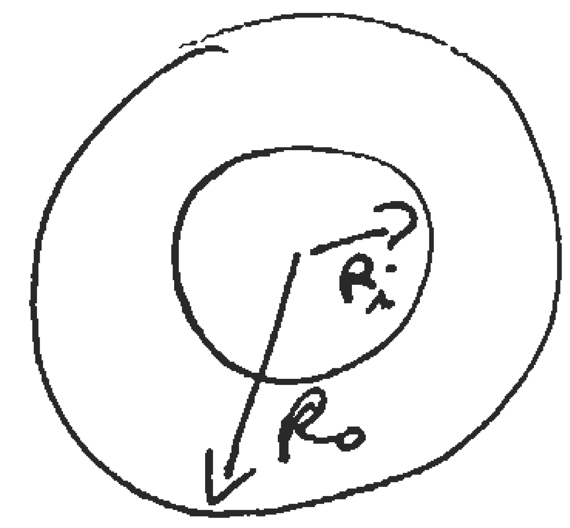
M14 $E = 70$ GPa



$R_i = \frac{4}{5} R_o$

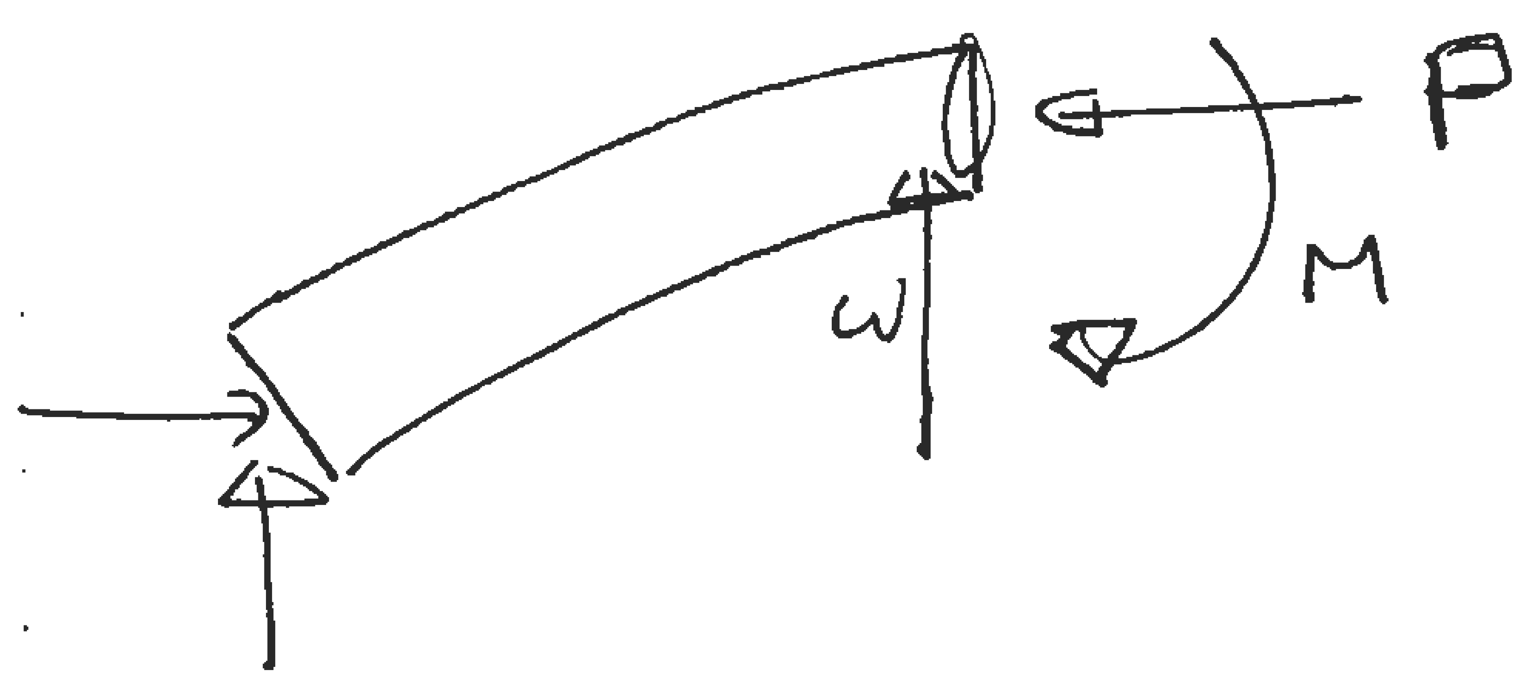
Basic Problem:

$\equiv$

which we met in class

$$\therefore\ w = e\left(\frac{1-\cos\sqrt{\frac{P}{EI}}\,L}{\sin\sqrt{\frac{P}{EI}}\,L}\sin\sqrt{\frac{P}{EI}}\,x + \cos\sqrt{\frac{P}{EI}}\,x - 1\right)$$

The stress at a given point



$$\sigma = \underset{\text{axial}}{\frac{P}{A}} \mp \underset{\text{bending}}{\frac{Mz}{I}}$$

and $M = EI\dfrac{d^2w}{dx^2}$

$$\frac{d^2w}{dx^2} = -e\left(\frac{P}{EI}\right)\left[\frac{1-\cos\sqrt{\frac{P}{EI}}\,L}{\sin\sqrt{\frac{P}{EI}}\,L}\sin\sqrt{\frac{P}{EI}}\,x + \cos\sqrt{\frac{P}{EI}}\,x\right] -$$

moment will be a maximum at $x = \frac{L}{2}$ since $w =$ max.

$$M = EI\frac{d^2w}{dx^2}$$

$$M_{max} = -ep\left[\frac{1-\cos\sqrt{\frac{P}{EI}}L}{\sin\sqrt{\frac{P}{EI}}L}\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\right]$$

let $\sqrt{\frac{P}{EI}}\frac{L}{2} = \theta$

$\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$M_{max} = -ep\left[\frac{1-(\cos^2\theta - \sin^2\theta)}{2\cancel{\sin\theta}\cos\theta}\cancel{\sin\theta} + \cos\theta\right]$$

$1 = \cos^2\theta + \sin^2\theta$

$$M_{max} = -Pe\left[\frac{\cancel{\cos^2\theta} + \sin^2\theta - \cancel{\cos^2\theta} + \sin^2\theta}{2\cos\theta} + \cos\theta\right]$$

$$= -Pe\left[\frac{\sin^2\theta}{\cos\theta}\cancel{\tan\theta} + \cos\theta\right]$$

$$= -Pe\left[\frac{\sin^2\theta + \cos^2\theta}{\cos\theta}\right] = -Pe\left[\frac{1}{\cos\theta}\right] \quad !$$

$$\therefore M_{max} = -Pe\left[\sec\sqrt{\frac{P}{EI}}\frac{L}{2}\right] \Leftarrow$$

max for tensile stress

$$\text{Max stress} = \frac{P}{A} \pm \frac{MZ}{I}$$

Let $\frac{4}{5} = \alpha$

$R_i = \alpha R_o$

$$A = \pi(R_o^2 - R_i^2) = \pi R_o^2(1-\alpha^2)$$

$$I = \frac{\pi}{4}(R_o^4 - R_i^4) = \frac{\pi R_o^4}{4}(1-\alpha^4)$$

$Z = R_o$, $e = R_o$

$$\therefore\ \sigma = \frac{P}{\pi R_o^2(1-\alpha^2)} + \frac{P R_o}{\pi R_o^{4}(1-\alpha^4)}\left[\sec\sqrt{\frac{P}{EI}}\,\frac{L}{2}\right]$$

$$\sigma = \frac{P}{\pi R_o^2}\left[\frac{1}{(1-\alpha^2)} + \frac{1}{R_o(1-\alpha^4)}\sec\sqrt{\frac{P}{EI}}\,\frac{L}{2}\right]$$

Need to iterate to solve. Calculate P_{crit} first $= 20\,\text{kN}$.

$$I = \frac{\pi\times(25\times10^{-3})^4}{4}\left(1-\left(\tfrac{4}{5}\right)^4\right) = 181.1\times10^{-9}$$

$EI = 12.7\times10^3$ $\qquad \frac{L}{2} = 1.25\,\text{m}$ $\qquad \frac{1}{25\times10^{-3}\left(1-\left(\frac{4}{5}\right)^4\right)} = 67.8$

$P = 10\,\text{kN}.\quad \sigma = 792\times10^6\,\text{Pa}$! too high $\qquad \frac{1}{1-\alpha^2} = \frac{25}{9}$

$P = 1\,\text{kN}\quad \sigma = 38.2\,\text{MPa}$

$P = 3\,\text{kN}\quad \sigma = 131\,\text{MPa}$ $\qquad \pi R_o^2 = 1.96\times10^{-3}$

$P = 2\,\text{kN}\quad \sigma = 81.5\,\text{MPa}$ $\qquad \frac{1}{\sqrt{EI}}\frac{L}{2} = 11.10\times10^{-3}$

$\therefore$ max load $\simeq 2\,\text{kN}$

For perfect column $\quad P_{crit} = \frac{\pi^2 EI}{L^2}$

$$= \frac{\pi^2\times12.7\times10^3}{2.5^2} = 20\times10^3$$