M11 Torsion of circular cross-section shafts

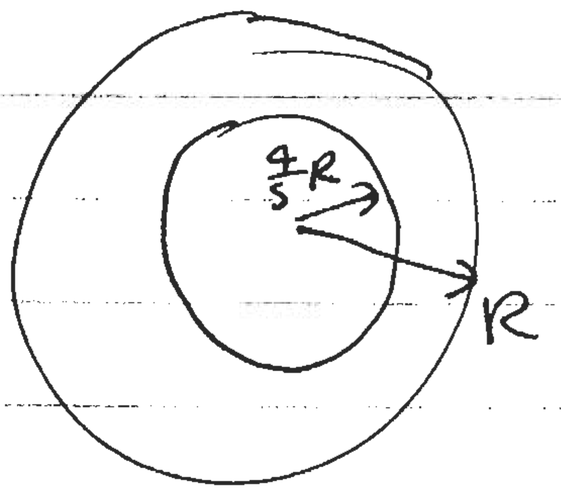
a) Torque shear stress relation: $\hat{\tau} = \frac{Tr}{J}$

for solid circular cross section $J = \frac{\pi R^4}{2}$

$$\therefore R^3 = \frac{2TR}{\pi \hat{\tau}_y} \Rightarrow R = \sqrt[3]{\frac{2T}{\pi \hat{\tau}_y}} = \sqrt[3]{\frac{2 \times 200 \times 10^3}{\pi \times 200 \times 10^6}}$$

$= 0.086\,m \Rightarrow$ diameter $= 0.172\,m$ ∈

b) Hollow shaft



$$J = \frac{\pi R^4}{2} - \frac{\pi}{2}\left(\frac{4}{5}\right)^4 R^4$$

$$= \frac{\pi R^4}{2}\left(1 - \left(\frac{4}{5}\right)^4\right)$$

$$= \frac{\pi R^4}{2}\left(1 - \frac{256}{625}\right)$$

$$\therefore \frac{\pi R^4}{2}(0.5904) = \frac{TR}{\hat{\tau}_y}$$

$$R = \sqrt[3]{\frac{2T}{\pi(0.5904)\hat{\tau}_y}} = \sqrt[3]{\frac{2 \times 200 \times 10^3}{\pi \times 0.5904 \times 200 \times 10^6}} = 0.103\,m$$

$\therefore$ diameter $= 0.205\,m$. ∈

Fractional weight $= \dfrac{\cancel{\pi}\left(0.086^2 - (0.103)^2\left(1-\left(\frac{4}{5}\right)^2\right)\right)}{\cancel{\pi}(0.086)^2}$

$$= 1 - \left(\frac{0.103}{0.086}\right)^2\left(1-\left(\frac{4}{5}\right)^2\right)$$

$$= 0.484 = 48.4\% \text{ weight saving.}$$

c) Twist angle. from $T = GJ\dfrac{d\phi}{dx}$

Since Torque is constant, twist angle $= L\dfrac{d\phi}{dx}$

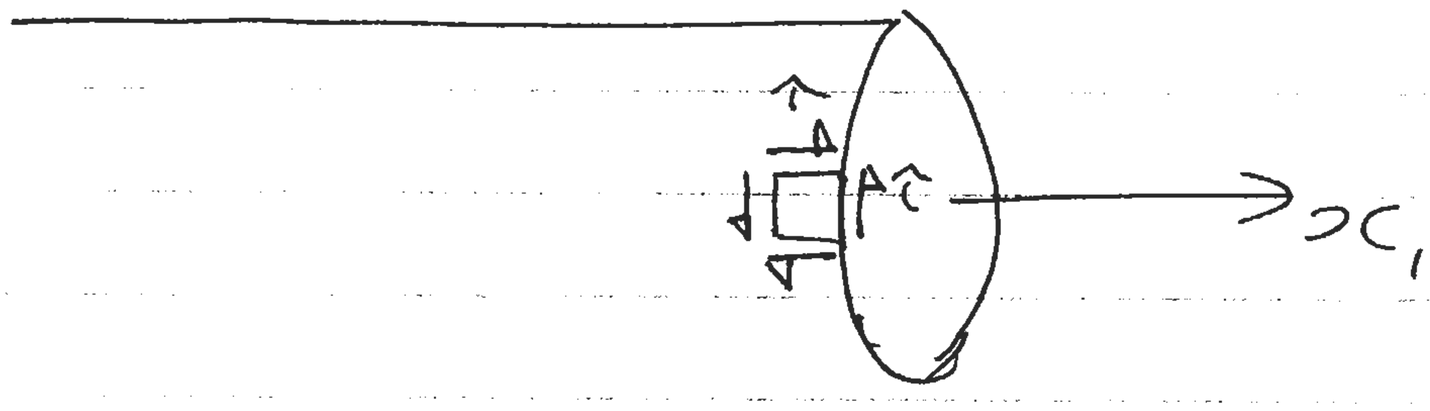
∴ ratio of twist angles is ratio of J

$$= \frac{\cancel{\pi}(0.103)^4}{\cancel{2}}\left(1-\frac{256}{625}\right) \times \frac{\cancel{2}}{\cancel{\pi}(0.086)^4}\Bigg($$

$$= \frac{(0.103)^4}{(0.086)^4}\left(1-\frac{256}{625}\right) = 1.215$$

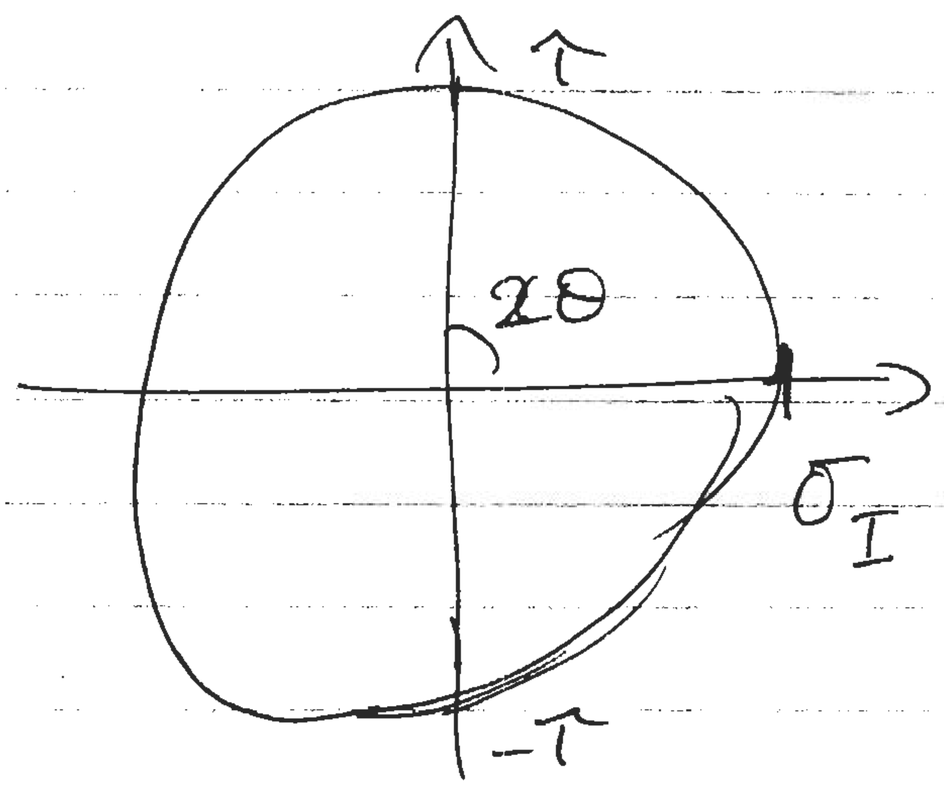
i.e. shaft (a) will twist 21.5% more than shaft (b).

d.



in case b (or a) shear stress τ acts in plane of section.

Drawing Mohrs circle



max tensile stress, σ_I will act at 45° to max shear i.e.