For any taper r: $\quad c(y) = c_{avg}\frac{2}{1+r}\left[1-(1-r)\frac{2y}{b}\right]$

a) Assuming $q \sim c$: $\quad \boxed{q(y) = q_{avg}\frac{2}{1+r}\left[1-(1-r)\frac{2y}{b}\right]}$

Total lift on half span: $F = q_{avg}\cdot\frac{b}{2} = 10\,N \;\rightarrow\; \boxed{q_{avg} = \frac{10N}{1m} = 10\,N/m}$

$$\boxed{S(y) = \int_{b/2}^{y} q(y)\,dy = q_{avg}\frac{2}{1+r}\left[y-(1-r)\frac{y^2}{b}\right]_{b/2}^{y} = q_{avg}\frac{2}{1+r}\left[y-\frac{b}{2}+\frac{1-r}{b}\left(\frac{b^2}{4}-y^2\right)\right]}$$

$$M(y) = \int_{b/2}^{y} S(y)\,dy = q_{avg}\frac{2}{1+r}\left[\frac{1}{2}y^2-\frac{b}{2}y+(1-r)\left(\frac{b^2}{4}y-\frac{1}{3}y^3\right)\right]_{b/2}^{y}$$

$$\boxed{M(y) = q_{avg}\frac{2}{1+r}\left[\frac{1}{2}\left(y^2-\frac{b^2}{4}\right)+\frac{b}{2}\left(\frac{b}{2}-y\right)+\left(\frac{1-r}{b}\right)\left(\frac{b^2}{4}\left(y-\frac{b}{2}\right)+\frac{1}{3}\left(\frac{b^3}{8}-y^3\right)\right)\right]}$$

could simplify this I suppose. Plots attached

b) $M = Ph = Pc\tau \;\rightarrow\; P(y) = \pm\frac{M(y)}{c(y)\tau}$, Plots attached.

c) $P = A\sigma \;\rightarrow\; A_{min}(y) = \frac{P(y)}{\sigma_{max}}$ same plot as $P(y)$, aside from scale

Area is roughly parabolic. $\quad Vol = \int_0^{b/2} A(y)\,dy \approx \frac{1}{3}A(0)\cdot\frac{b}{2}$

$$Vol \simeq \frac{1}{3}\frac{P(0)}{\sigma_{max}}\cdot\frac{b}{2} = \frac{b}{6}\frac{1}{\sigma_{max}}\frac{M(0)}{c(0)}\frac{1}{\tau} \quad \text{(one cap for half-wing)}$$

we have $M(0) = q_{avg}\frac{2}{1+r}\left[-\frac{b^2}{8}+\frac{b^2}{4}+\frac{1-r}{b}\left(-\frac{b^3}{8}+\frac{b^3}{24}\right)\right] = q_{avg}\frac{2}{1+r}\left[\frac{b^2}{8}-(1-r)\frac{b^2}{12}\right]$

$c(0) = c_{avg}\frac{2}{1+r}$

$$\therefore Vol = \frac{b^3}{6}\frac{1}{\sigma_{max}\tau}\frac{q_{avg}}{c_{avg}}\left(\frac{1}{8}-\frac{1-r}{12}\right) = \begin{cases} 11.9\times10^{-6}\,m^3 = 11.9\,cm^3 & (r=1.0)\\ 7.9\times10^{-6}\,m^3 = 7.9\,cm^3 & (r=0.5)\end{cases}$$

4-cap mass $m = 4\rho\cdot Vol = \begin{cases} 6.0\,g & (r=1.0)\\ 4.0\,g & (r=0.5)\end{cases}$

d) $I = \frac{1}{2}Ah^2 = \frac{1}{2}Ac^2\tau^2 = \frac{1}{2}\frac{M}{c\tau\sigma_{max}}c^2\tau^2 = \frac{1}{2}\frac{Mc\tau}{\sigma_{max}}$

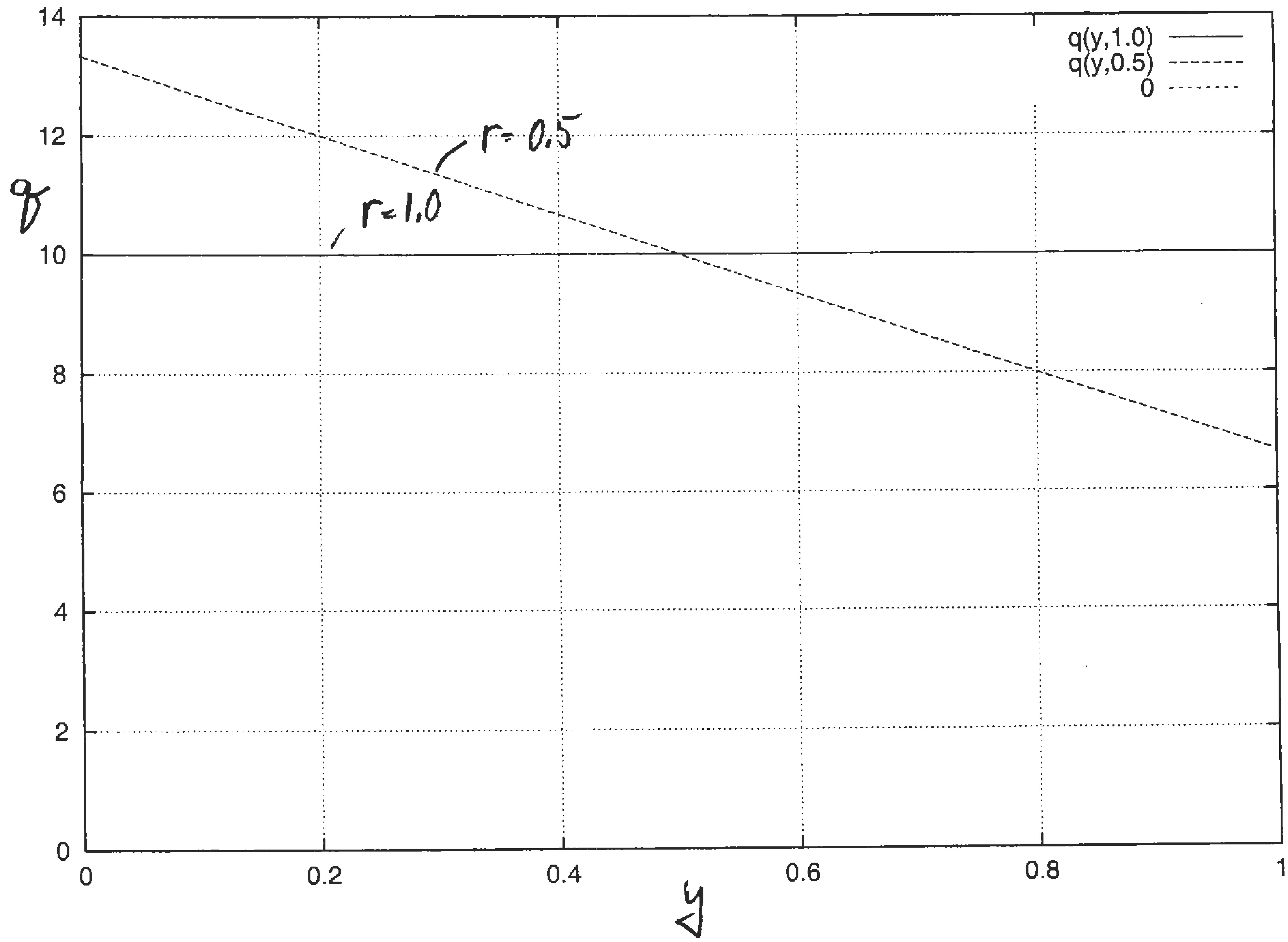
$K = \frac{M}{EI} = \frac{2\sigma_{max}}{E}\frac{1}{c\tau}$, $\quad K(0) = 2\,\frac{7\,MPa}{1.36\,GPa}\cdot\frac{1}{0.08}\,\frac{1}{c(0)} = 0.129\cdot\frac{1+r}{2c_{avg}} = \begin{cases} 0.52/m & r=1.0\\ 0.39/m & r=0.5\end{cases}$

$\delta = \frac{1}{2}K(b/2)^2 = \begin{cases} 0.258\,m & r=1.0\\ 0.193\,m & r=0.5\end{cases}$

The tapered wing seems better in all respects.
- better e (lower C_{D_i})
- lighter spar
- smaller δ

Also, balsa caps are very light. Look attractive.

M10



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q(y,0.5)
0
r=0.5
r=1.0
q
y
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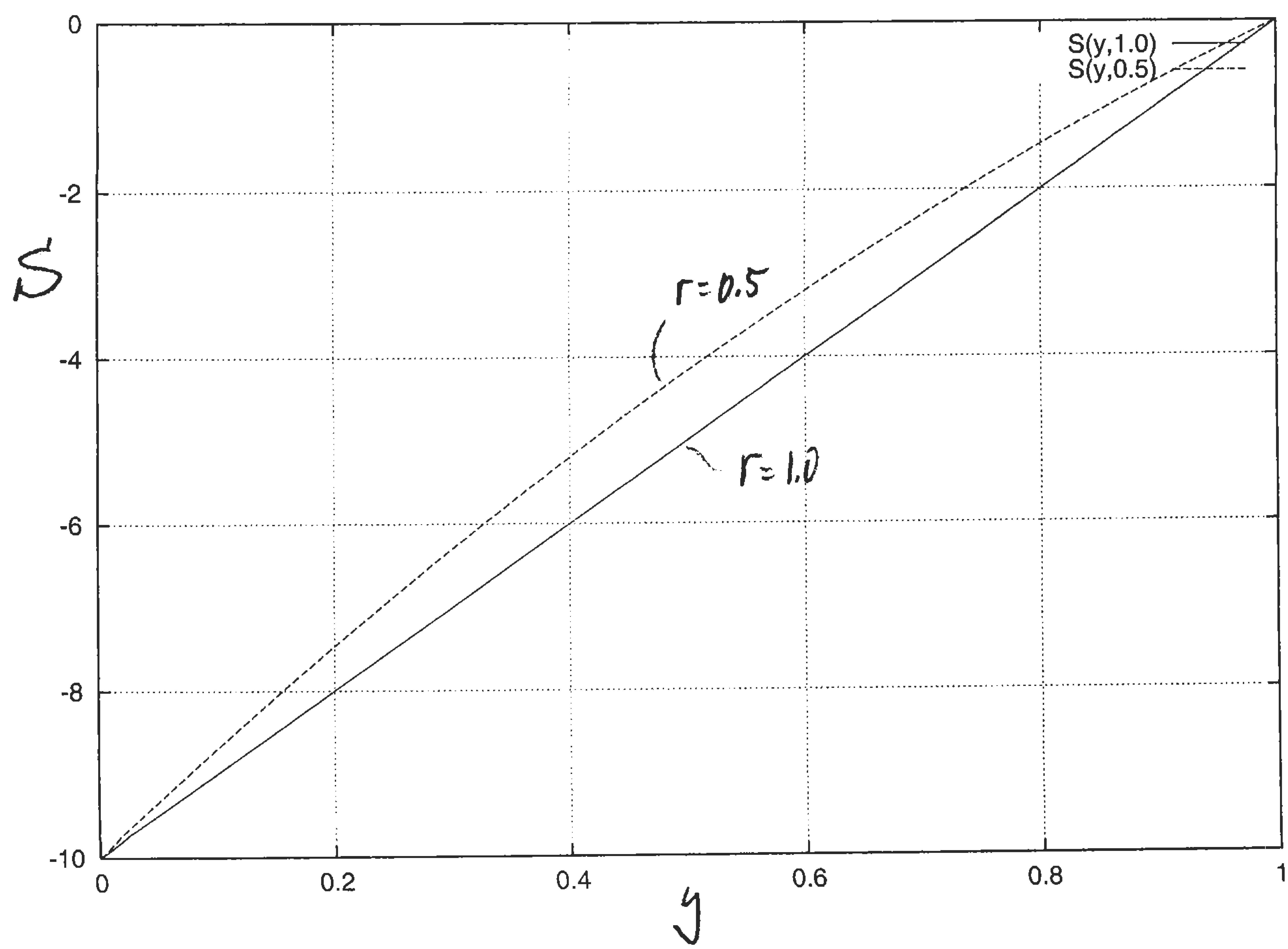
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1



S(y,1.0)
S(y,0.5)
r=0.5
r=1.0
S
y
0
-2
-4
-6
-8
-10
0.2
0.4
0.6
0.8
1

M10