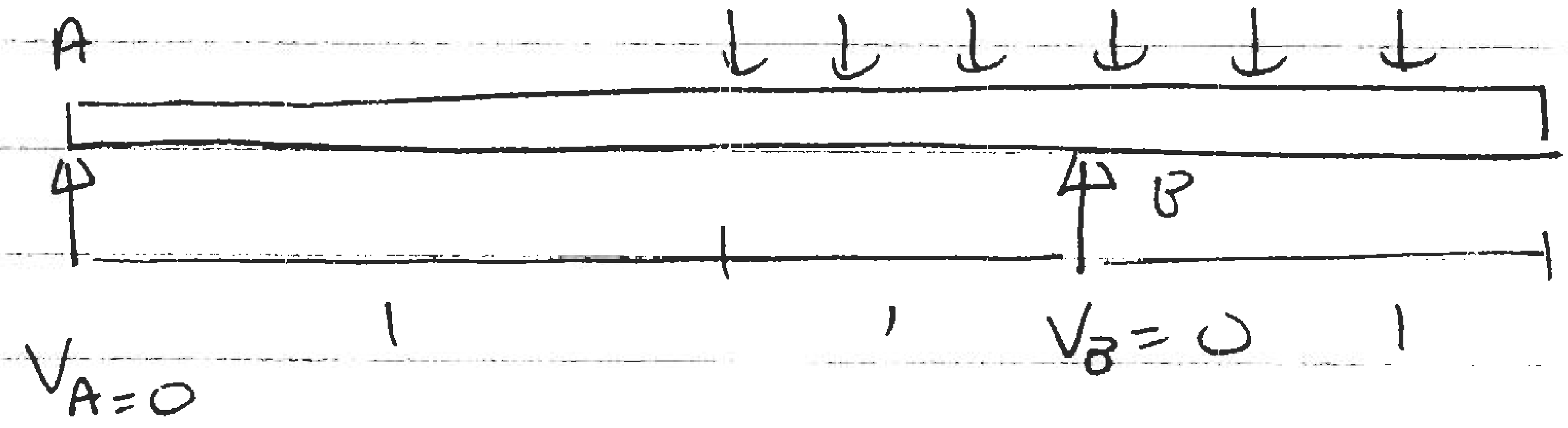
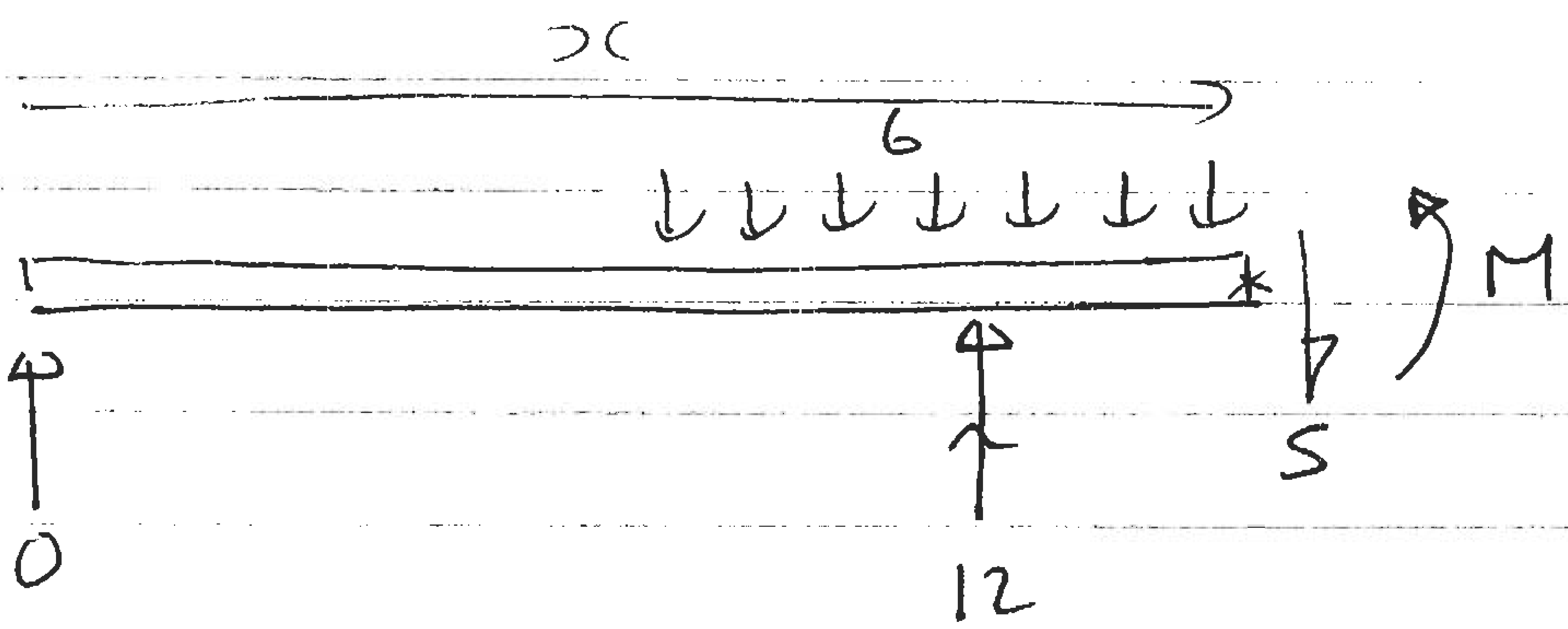
M7/M8

a)



Apply Macauley's method



$$\circlearrowleft \sum M_x = 0: \quad M + 6\frac{\{x-1\}^2}{2} - 12\{x-2\} = 0$$

$$M = -\frac{6\{x-1\}^2}{2} + 12\{x-2\}$$

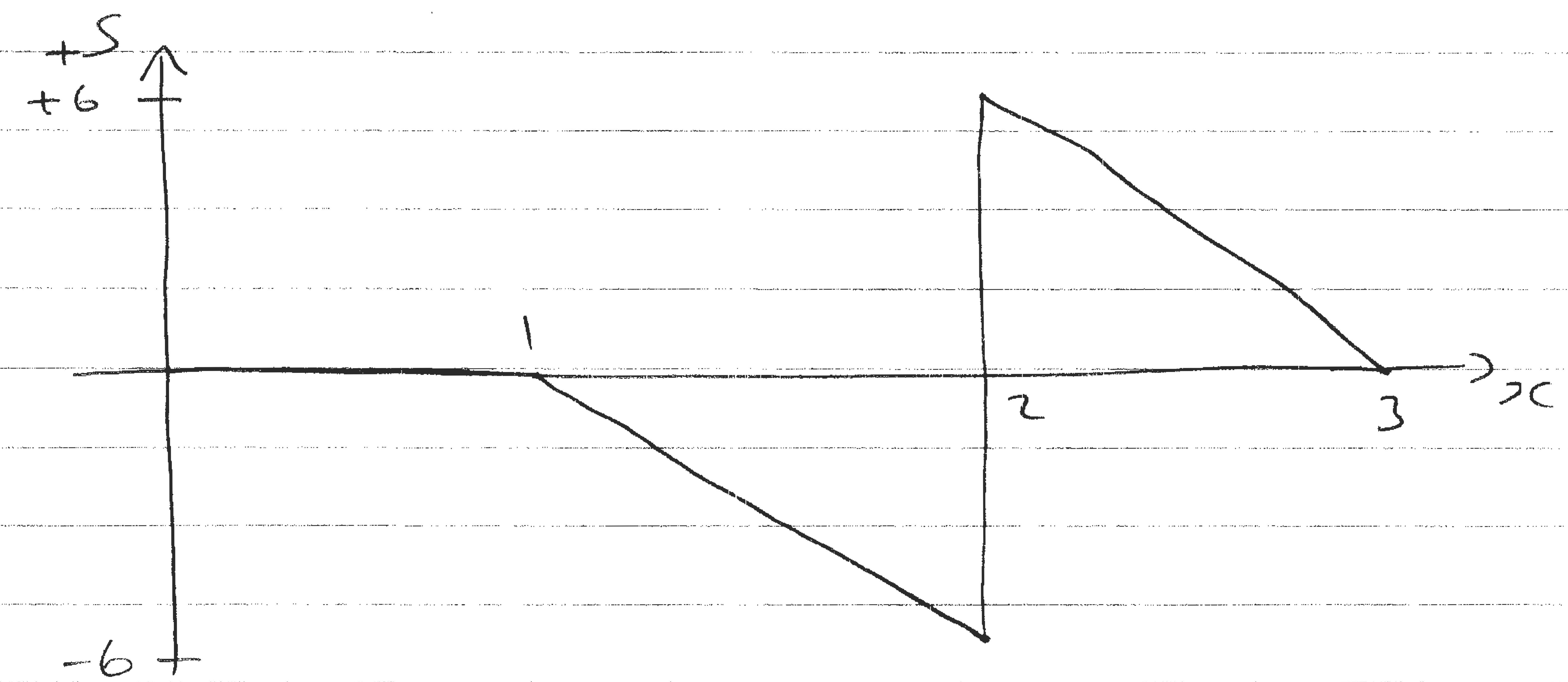
$$\cancel{\sum F_z \uparrow = 0:}$$

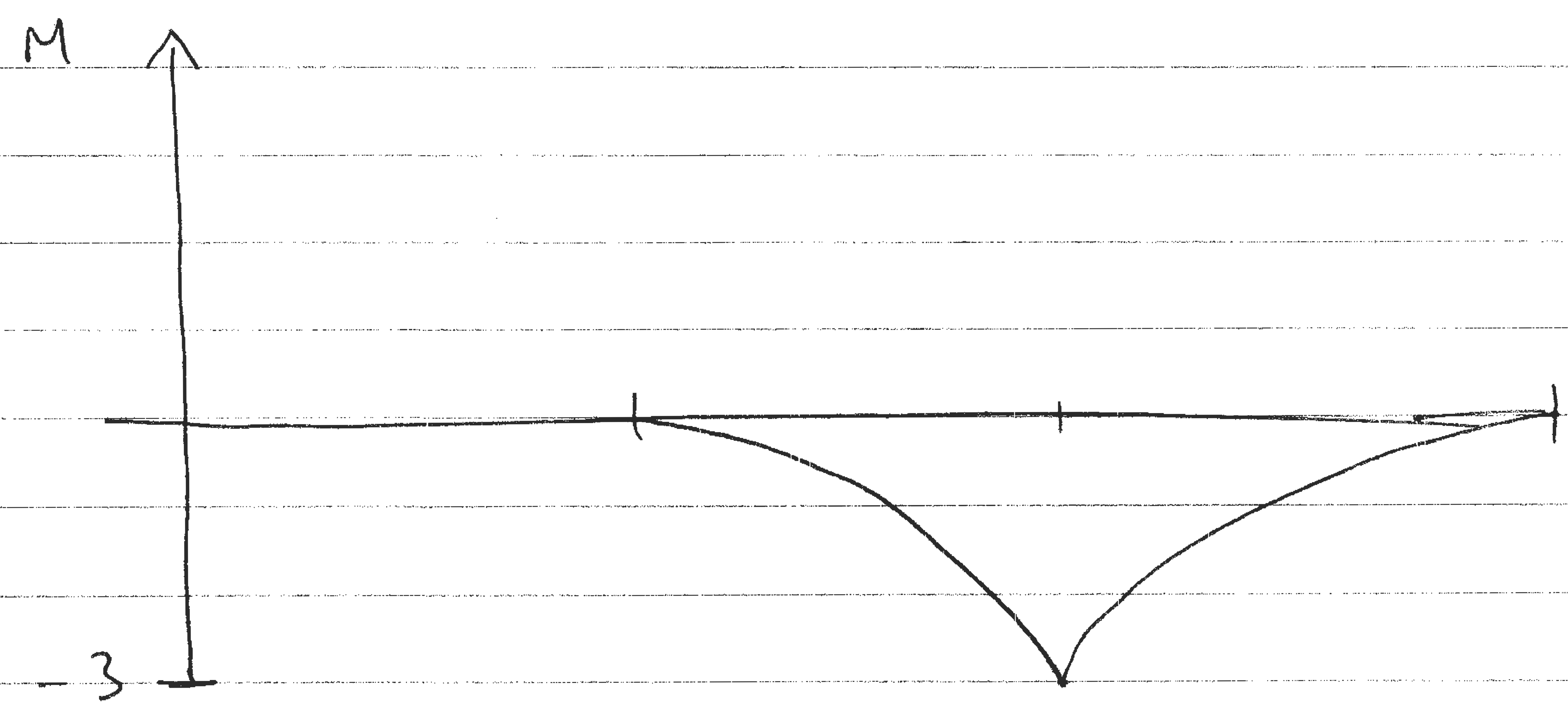
$$\cancel{-S - 6\{x-1\} + 12 = 0}$$

$$\cancel{S = -6\{x-1\} + 12}$$

from $\cancel{M = -EI\frac{d^2y}{dx^2}}$ $\quad S = \frac{dM}{dx}$

$$S = \frac{dM}{dx} = +6\{x-1\} + 12\{x-2\}^0$$





6)

$$EI\frac{d^2w}{dx^2} = -3\{x-1\}^2 + 12\{x-2\}$$

$$EI\frac{dw}{dx} = -\frac{3\{x-1\}^3}{3} + \frac{12\{x-2\}^2}{2} + A$$

$$EIw = -\frac{\{x-1\}^4}{4} + \frac{\cancel{12}6\{x-2\}^3}{3} + Ax + B$$

apply boundary conditions

$w = 0$ @ $x = 0$, $x = 2$.

@ $x = 0 \Rightarrow B = 0$ $\qquad \{x-1\}, \{x-2\} = 0$

@ $x = 2$: $-\frac{1}{4} + A \cancel{x} 2 = 0$

$$\therefore A = \frac{1}{8}$$

$$EIw = -\frac{\{x-1\}^4}{4} + 2\{x-2\}^3 + \frac{x}{8} \Leftarrow$$

Maximum deflection occurs either

for $0 < x < 2$ or at $x = \cancel{4}\ 3$

for $0 < x < 1$ $\qquad EI\dfrac{dw}{dx} = \cancel{A}\,\dfrac{1}{8}$ $\qquad \therefore$ no min/max

for $1 < x < 2$

$$EI\frac{dw}{dx} = -\cancel{3}\,\{x-1\}^3 + \cancel{A}\,\frac{1}{8}$$

$$-x^3 + 3x^2 - 3x + \frac{9}{8} \cancel{+\frac{1}{8}} = 0$$

Solution $x = 1.04$.

$$\Rightarrow EIw = \frac{-(0.04)^4}{4} + \frac{1.04}{8} = +0.13\,\cancel{EI} \Leftarrow$$

$$w = \frac{0.13}{EI} \Leftarrow \qquad @\ x = 1.04\,\text{m}.$$

This is the maximum up bending

$$w = \frac{0.13\times 10^{3}}{3.54\times 10^{6}} = 36.7\,\mu\text{m} \Leftarrow$$

kNm

for $x = 3$

$$EIw = -\frac{(2^4)}{4} + 2(1)^3 + \frac{3}{8}$$

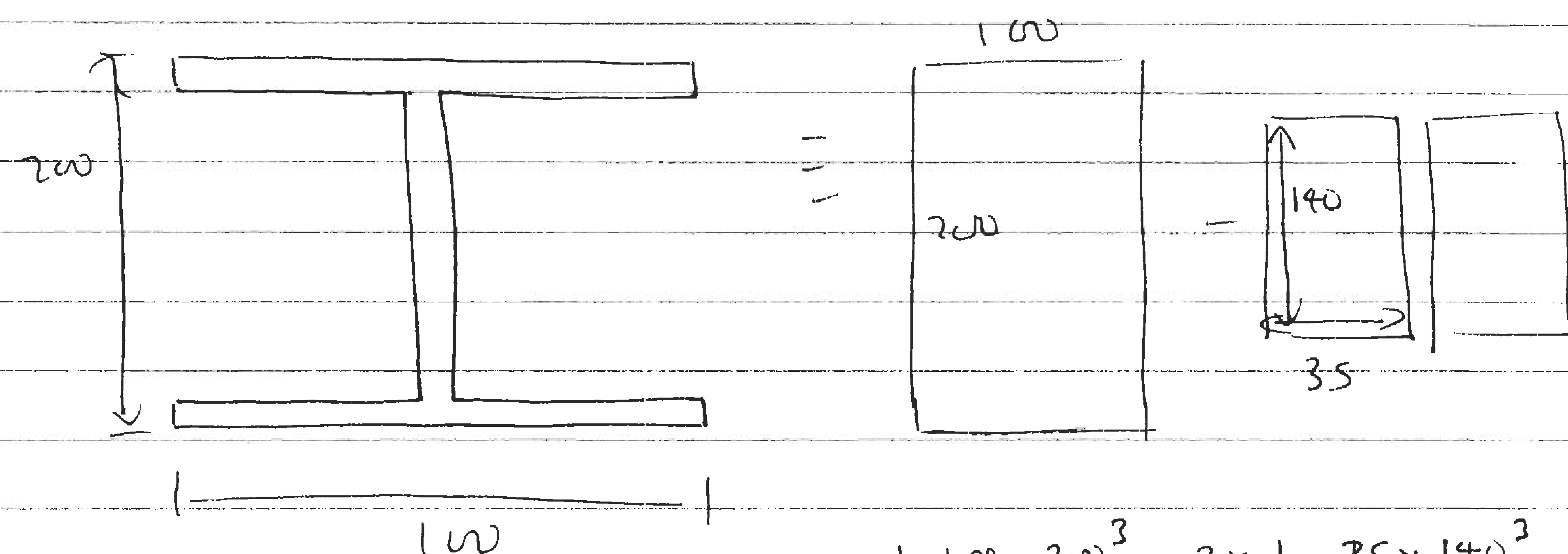
$$EIw = -4 + 2 + \frac{3}{8} \qquad w = \frac{-13}{8EI}$$

$$= \frac{-1.625}{EI}$$

This is the maximum down bending @ $x = 3$ m.

$w = -459\ \mu m$. (0.459 mm.)

Calculation of I



$$= \frac{1}{12}\, 100 \times 200^3 - 2 \times \frac{1}{12} \times 35 \times 140^3$$

$$= 50.7 \times 10^6\ mm^4$$

$$EI = 70 \times 10^9 \times 50.7 \times 10^6 \times 10^{-12} = 3.54 \times 10^6\ Nm^2$$

$$= -459$$

c)

$$\sigma_{xx} = -\frac{Mz}{I}$$ maximum at max bending moment, max z @ $x = 2$, $M = 3$ kNm @ $\frac{h}{2} = \pm 100$ mm

$$\therefore \sigma_{xx} = \frac{3\times10^{3} \times 100 \times 10^{-3}}{50.7\times10^{6} \times 10^{-12}} = 5.9 \text{ MPa}$$

Now no danger of yield due to tensile stresses

$$\sigma_{xz} = -\frac{SQ}{Ib}. \qquad Q = \int_{z}^{h/2} z\, dA$$

will be maximum at center of beam. i.e. for $z = 0$

$$Q = \int_0^{70} z \,.\, 30\, dz + \int_{70}^{100} z \,.\, 100\, dz$$

$$= \left[\frac{30 z^2}{2}\right]_0^{70} + \left[\frac{\cancel{50}\cancel{0}\, z^2}{2}\right]_{70}^{100} = 328.5 \times 10^{3} \text{ mm}^{3}$$

$$\therefore \sigma_{xz} = -\frac{6\times10^{3} \times 328.5\times10^{3} \times \cancel{10^{-9}}}{50.7\times\cancel{10^{6}} \cancel{\times10^{-12}} \times 30\times\cancel{10^{-3}}} = 1.29\times10^{6} \text{ MPa}$$

Note I beam shear stress ≈ bending stress

Yield is not a problem.